Name_____ UIN_____

1-11	/55	14	/10
12	/20	15	/10
13	/10	Total	/105

MATH 171 Final Exam Fall 2021

Sections 503 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. For what value(s) of p are the vectors $\vec{a} = (3, p)$ and $\vec{b} = (4, 6)$, perpendicular?

a. $\frac{1}{2}$ only

b. $-\frac{1}{2}$ only

c. 2 only

d. -2 only

e. 0 only

f. $\frac{1}{2}$ or $-\frac{1}{2}$ only

g. 2 or -2 only

h. 2 or $\frac{1}{2}$ only

i. -2 or $-\frac{1}{2}$ only

j. no value of p

2. Compute $\lim_{x\to 3} \frac{x^2 - 4x + 3}{x^2 - 9}$

a. $-\frac{1}{9}$

b. $-\frac{1}{3}$

c. $-\frac{1}{6}$

d. 0

e. $\frac{1}{6}$

f. $\frac{1}{3}$

g. $\frac{1}{2}$

h. $\frac{2}{3}$

i. 1

j. undefined

3. As $x \to \infty$, the function $f(x) = \sqrt{x^2 + 5x} - \sqrt{x^2 + 2x}$ has a horizontal asymptote at

a. $y = -\infty$

b. $y = -3$

c. $y = -\frac{3}{2}$

d. $y = -\frac{1}{2}$

e. $y = 0$

f. $y = \frac{1}{2}$

g. $y = \frac{3}{2}$

h. $y = 1$

i. $y = 3$

j. $y = \infty$

4. The limit $\lim_{h\to 0} \frac{4(2+h)^3 - 32}{h}$ can be interpreted as which of the following?

a. $f'(2)$ where $f(x) = x^3$
b. $f'(2)$ where $f(x) = 4x^3$
c. $f'(2)$ where $f(x) = x^4$
d. $f'(2)$ where $f(x) = 12x^2$
f. $f'(4)$ where $f(x) = x^3$
g. $f'(4)$ where $f(x) = 4x^3$
h. $f'(4)$ where $f(x) = x^4$
i. $f'(4)$ where $f(x) = 12x^2$
i. $f'(32)$ where $f(x) = x^3$
j. $f'(32)$ where $f(x) = 4x^3$
k. $f'(32)$ where $f(x) = x^4$
l. $f'(32)$ where $f(x) = 12x^2$

5. Find the line tangent to $y = \frac{1}{4}x^4$ at $x = 2$. Its y-intercept is:

a. -20
b. -16
c. -12
d. -4
e. 0
f. 4
g. 12
h. 16
i. 20
j. none of these

6. A spacecraft is being sent to Mars. Its distance from the earth is given by $p(t) = 7t^3 + 1$. At time $t = 2$ the position is measured, but the error in the time measurement is ± 0.1. What is the resulting error in the calculated position?

a. ± 7.3
b. ± 7.4
c. ± 8.4
d. ± 8.5
e. 0
f. ± 73
g. ± 74
h. ± 84
i. ± 85
j. Impossible to determine.

7. Find the line tangent to the curve $y^3 = x^2 - xy$ at $(x,y) = (-2,2)$. Its y-intercept is:

a. $-\frac{3}{5}$
b. $-\frac{4}{5}$
c. $-\frac{5}{4}$
d. $-\frac{5}{3}$
e. 0
f. $\frac{3}{5}$
g. $\frac{4}{5}$
h. $\frac{5}{4}$
i. $\frac{5}{3}$
j. 1

8. Compute $\lim_{x\to 0} \frac{\sin x - x}{x^3}$

a. $-\frac{1}{6}$

b. $-\frac{1}{3}$

c. $-\frac{1}{2}$

d. 0

e. $\frac{1}{6}$

f. $\frac{1}{3}$

g. $\frac{1}{2}$

h. undefined

9. Find the area under $y = \sqrt{x}$ between $x = 1$ and $x = 4$.

a. 1

b. 3

c. 4

d. 6

e. 12

f. $\frac{14}{3}$

g. $\frac{16}{3}$

h. $\frac{28}{3}$

i. $\frac{21}{2}$

j. $\frac{21}{4}$

10. Compute $\int_0^{\pi} e^{\cos x} \sin x \, dx$

a. 0

b. $\frac{1}{e} - e$

c. $e - \frac{1}{e}$

d. $-\frac{1}{e}$

e. $-e$

f. 1

g. $1 - e$

h. $e - 1$

i. $1 - \frac{1}{e}$

j. $\frac{1}{e} - 1$

11. Calculate $\lim_{x\to 0^+} \frac{1}{x} \int_0^x e^{t^2} \, dt$

a. $-\infty$

b. $-e$

c. -1

d. $-\frac{1}{e}$

e. 0

f. $\frac{1}{e}$

g. 1

h. e

i. e^2

j. ∞

Work Out: (Points indicated. Part credit possible. Show all work.)

12. (20 points) Consider the function $g(x) = \frac{3}{3+x^2}$. Find each of the following and say why. (If an item does not exist, say NONE and say why not.) Then graph the function.

a. horizontal asymptote as $x \to \infty$:

b. horizontal asymptote as $x \to -\infty$:

c. $g'(x)$ and critical points:

d. Intervals where g is increasing and decreasing:

e. $g''(x)$ and secondary critical points

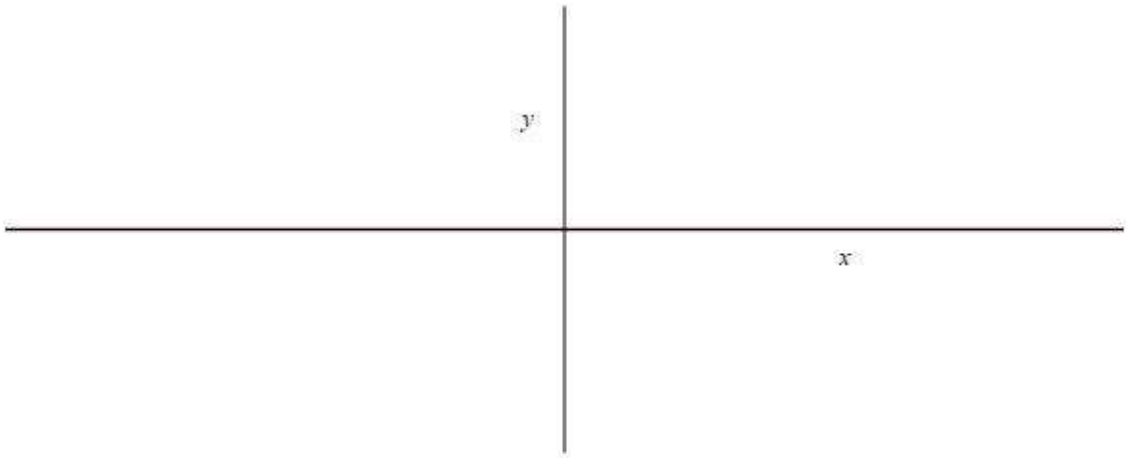
f. Intervals where g is concave up and down:

g. x-coordinate at each local maximum.

h. x-coordinate at each local minimum

i. x-coordinate at each inflection point

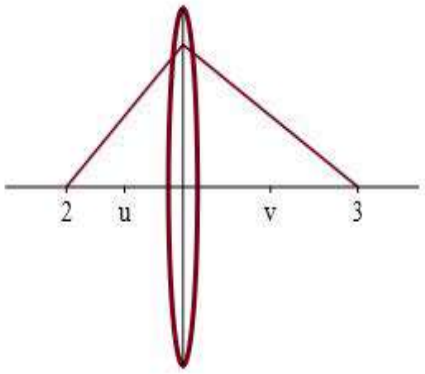
j. Plot: (Put an $\times$ at each local minimum or maximum. Put an $\bigcirc$ at each inflection point.)



13. (10 points) Find the equation of the line tangent to $y = x^2$ at the general point $x = a$. For what value(s) of a does the tangent line pass through the point $(3, 8)$?

14. (10 points) When light passes through a lens with focal length f the distance to the object, u, is related to the distance to the image, v, by the equation

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$



Here f is a constant. As u changes, v changes.

If $f = \frac{6}{5}$, $u = 2$, and $\frac{du}{dt} = -0.4$, find v and $\frac{dv}{dt}$.

Is v getting longer or shorter?

15. (10 points) A rectangle is inscribed in the upper half of the ellipse $\frac{x^2}{4} + y^2 = 1$ with its base on the x-axis. Find the maximum area of such a rectangle.