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MATH 171 **Final Exam** **Spring 1998**

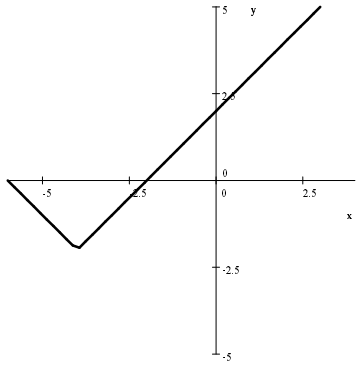
Section 501 **P. Yasskin**

Part I: Multiple Choice (3 points each) No Partial Credit

1. Find the angle between the vectors $\vec{a} = \langle \sqrt{2}, 1, 1 \rangle$ and $\vec{b} = \langle \sqrt{2}, 1, -1 \rangle$

a. 0°
b. 30°
c. 45°
d. 60°
e. 90°

2. The plot at the right is the graph of the function $f(x) =$



a. $|x-4|-2$
b. $|x-4|+2$
c. $|x+4|-2$
d. $|x+4|+2$

3. $\lim_{x \to 4} \dfrac{2\sqrt{x} - x}{x-4} =$

a. $-\dfrac{1}{2}$
b. $-\dfrac{1}{4}$
c. 0
d. $\dfrac{1}{3}$
e. $\dfrac{1}{2}$

4. For what value of p is the function $f(x) = \begin{cases} 2x^2 + 3 & \text{for } x \leq 1 \\ x + p & \text{for } x > 1 \end{cases}$ continuous?

a. 0
b. 1
c. 2
d. 3
e. 4

5. Find the equation of the line tangent to the curve $y = x^4 - 2x^2 - 4$ at $x = 2$.

a. $y = 24x - 44$
b. $y = 4x + 16$
c. $y = 24x + 4$
d. $y = 4x - 4$
e. $y = 24$

6. If $f(x) = \frac{e^x + 1}{e^x - 1}$ then $f'(x) =$

a. $\frac{2e^{2x}}{(e^x - 1)^2}$
b. $\frac{2e^{2x}}{(e^x + 1)^2}$
c. $\frac{-2e^x}{(e^x - 1)^2}$
d. $\frac{-2e^x}{(e^x + 1)^2}$
e. $\frac{2e^x}{(e^x + 1)^2}$

7. $\frac{d}{dx}[\ln(\cos x)] =$

a. $\sec x$
b. $\tan x$
c. $\cot x$
d. $-\cot x$
e. $-\tan x$

8. A rocket is fired straight up and its altitude is given by $y = t^3 - t^2$. Find its acceleration at $t = 2$.

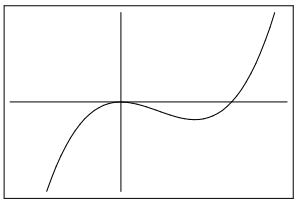
a. 2
b. 4
c. 6
d. 8
e. 10

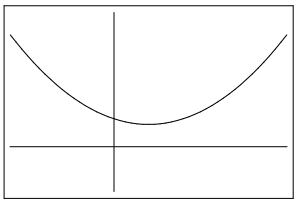
9. The figure consists of 5 squares of equal area. The area of the whole figure is 405 square centimeters. Find the length of one side of the square.

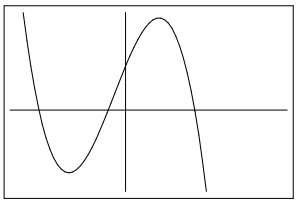
a. 9
b. 81
c. 5
d. 3
e. 1

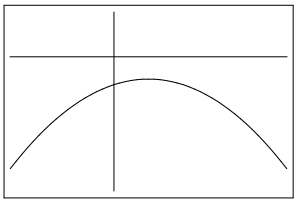
10. The acceleration of an object moving in a straight line can be determined from
a. the slope of the distance-time graph.
b. the area below the distance-time graph.
c. the slope of the velocity-time graph.
d. the area below the velocity-time graph.

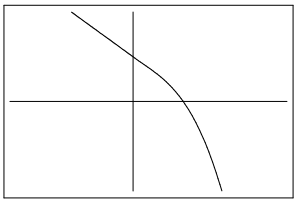
11. Which of the following graphs has these features: $f'(0) > 0, \quad f'(1) < 0,$ and $f''(x)$ is always negative?

a. 

b. 

c. 

d. 

e. 

12. If $y = \sqrt{x}^{\,x}$ then $\frac{dy}{dx} =$
a. $x\sqrt{x}^{\,x-1}$
b. $\sqrt{x}^{\,x}\left(\frac{1}{2} + \frac{1}{2}\ln x\right)$
c. $\sqrt{x}^{\,x}\ln\sqrt{x}$
d. $x\sqrt{x}^{\,x-1}\left(\frac{1}{2\sqrt{x}}\right)$
e. $\sqrt{x}^{\,x}\left(\frac{1}{2\sqrt{x}}\right)$

13. A rocket starts at rest, is fired straight up and its acceleration is given by $a = 200e^{-t}$ miles/hour^2. Find its velocity at $t = 2$ hours.

a. 200

b. $200 - \frac{200}{e^2}$

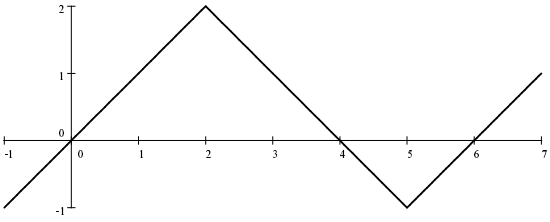
c. $-\frac{200}{e^2}$

d. $\frac{200}{e^2}$

e. $\frac{400}{e^2}$

14. The plot at the right is the graph of a function $y = f(x)$.

Compute $\int_0^6 f(x)\,dx$.



a. 1

b. 2

c. 3

d. 4

e. 5

15. Find the area under the curve $y = \frac{4}{3}x^3 + x^2$ above the x-axis, between $x = 1$ and $x = 3$.

a. $\frac{128}{3}$

b. 36

c. $\frac{110}{3}$

d. 48

e. $\frac{106}{3}$

Part II: Work Out Problems Partial credit will be given.
Calculators are permitted after the scantrons are collected.

16. (10 points) Suppose the function $y = f(x)$ is implicitly defined by the equation

$$x^2y - 2y^5 = x.$$

a. Find $\frac{dy}{dx}$ at the point $(x,y) = (2,1)$.

b. Find the equation of the line tangent to $y = f(x)$ at the point $(x,y) = (2,1)$.

17. (8 points) An igloo has the shape of a ***hemisphere*** of radius 5 feet. It is covered with snow which is 1 foot thick and melting at the rate of $\frac{1}{4}$ foot/hour. At what rate is the volume of snow decreasing?
Note: The volume of a ***sphere*** is $V = \frac{4}{3}\pi r^3$.

18. (7 points) Carbon-14 has a half-life of 5730 years. How long will it take 5 grams of Carbon-14 to decay to 3 grams?

19. (10 points) Find the absolute maximum and minimum values of the function

$f(x) = 2\sqrt{1-x^2} + \sin^{-1}x$ on the interval $[-1, 1]$.

Remember: The $\sin^{-1}x$ is given in radians.

20. (10 points) Find the dimensions and area of the largest rectangle that can be inscribed in a right triangle with legs of lengths 5 cm and 6 cm, if two sides of the rectangle lie along the legs.

21. (10 points) Compute the integral:

$$\int_0^1 \frac{x^2+1}{(x^3+3x+1)^3}\,dx =$$