Name________________________________

MATH 172 Final Exam Spring 2019

Sections 501 P. Yasskin

15 Multiple Choice: (4 points each. No part credit.)

1. Compute $\int 3x^2 \ln x\,dx$.

a. $6x\ln x - 6x + C$
b. $x^3\ln x - \dfrac{x^3}{3} + C$
c. $6x\ln x + 6x + C$
d. $x^3\ln x + \dfrac{x^3}{3} + C$
e. $\dfrac{x^3}{3}\ln x - \dfrac{x^3}{9} + C$

2. Compute $\int \sec^4\theta\,d\theta$.

a. $\dfrac{(\ln|\sec\theta + \tan\theta|)^5}{5} + C$
b. $\dfrac{\tan^5\theta}{5} - \dfrac{2\tan^3\theta}{3} + \tan\theta + C$
c. $\dfrac{\tan^5\theta}{5} + \dfrac{2\tan^3\theta}{3} + \tan\theta + C$
d. $\dfrac{\tan^3\theta}{3} - \tan\theta + C$
e. $\dfrac{\tan^3\theta}{3} + \tan\theta + C$

1-15	/60	17	/15
16	/15	18	/15
		Total	/105

3. Compute $\int \sqrt{4-x^2}\,dx$.

a. $\arcsin\frac{x}{2} + \frac{x}{3}(4-x^2)^{3/2} + C$
b. $2\arcsin\frac{x}{2} - x\sqrt{4-x^2} + C$
c. $2\arcsin\frac{x}{2} + \frac{x}{2}\sqrt{4-x^2} + C$
d. $\arcsin\frac{x}{2} - x\sqrt{4-x^2} + C$
e. $\arcsin\frac{x}{2} + x(4-x^2)^{3/2} + C$

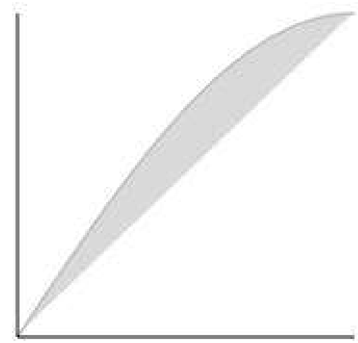
4. The integral $\int_1^\infty \frac{1}{x^3 + \sqrt[3]{x}}\,dx$.

a. converges by comparison to $\int_1^\infty \frac{1}{x^3}\,dx$.
b. diverges by comparison to $\int_1^\infty \frac{1}{x^3}\,dx$.
c. converges by comparison to $\int_1^\infty \frac{1}{\sqrt[3]{x}}\,dx$.
d. diverges by comparison to $\int_1^\infty \frac{1}{\sqrt[3]{x}}\,dx$.

5. Find the average value of the function $f(x) = \frac{1}{1+x^2}$ on the interval $[0, \sqrt{3}\,]$.

a. $\frac{\ln 4}{\sqrt{3}}$
b. $\frac{\ln 4}{2\sqrt{3}}$
c. $\frac{\pi}{6\sqrt{3}}$
d. $\frac{\pi}{3\sqrt{3}}$
e. $\frac{\pi}{2\sqrt{3}}$

6. The region between $y = \sin x$ and $y = \frac{2x}{\pi}$ for $0 \le x \le \frac{\pi}{2}$ is rotated about the y-axis. Which integral gives the volume swept out?



a. $V = \int_0^{\pi/2} 2\pi x\left(\frac{2x}{\pi} - \sin x\right) dx$

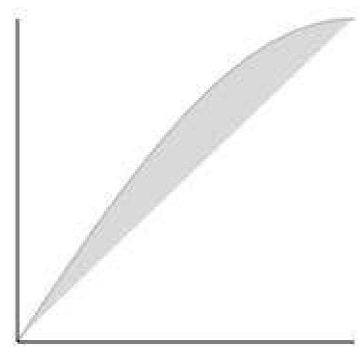
b. $V = \int_0^{\pi/2} 2\pi\left(\sin^2 x - \frac{4x^2}{\pi^2}\right) dx$

c. $V = \int_0^{\pi/2} 2\pi x\left(\sin x - \frac{2x}{\pi}\right) dx$

d. $V = \int_0^{\pi/2} \pi\left(\frac{4x^2}{\pi^2} - \sin^2 x\right) dx$

e. $V = \int_0^{\pi/2} \pi\left(\sin^2 x - \frac{4x^2}{\pi^2}\right) dx$

7. The region between $y = \sin x$ and $y = \frac{2x}{\pi}$ for $0 \le x \le \frac{\pi}{2}$ is rotated about the x-axis. Which integral gives the volume swept out?



a. $V = \int_0^{\pi/2} 2\pi x\left(\frac{2x}{\pi} - \sin x\right) dx$

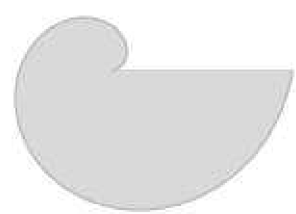
b. $V = \int_0^{\pi/2} 2\pi\left(\sin^2 x - \frac{4x^2}{\pi^2}\right) dx$

c. $V = \int_0^{\pi/2} 2\pi x\left(\sin x - \frac{2x}{\pi}\right) dx$

d. $V = \int_0^{\pi/2} \pi\left(\frac{4x^2}{\pi^2} - \sin^2 x\right) dx$

e. $V = \int_0^{\pi/2} \pi\left(\sin^2 x - \frac{4x^2}{\pi^2}\right) dx$

8. Find the area inside the first loop of the spiral $r = \theta$ for $0 \le \theta \le 2\pi$.



a. $2\pi^2$

b. $\frac{4\pi^3}{3}$

c. $\frac{4\pi^2}{3}$

d. $\frac{2\pi^2}{3}$

e. $\frac{8\pi^3}{3}$

9. Find the center of mass of a bar which is 6 cm long and has density $\delta = x + x^2$ where x is measured from one end.

a. $\frac{22}{5}$
b. $\frac{5}{22}$
c. $\frac{11}{5}$
d. $\frac{5}{11}$
e. $\frac{8}{5}$

10. The series $\sum_{n=0}^{\infty} \frac{(x-3)^n}{2^n(n^3 + \sqrt[3]{n}\,)}$ has radius of convergence $R = 2$. Find its interval of convergence.

a. $(1,5)$
b. $[1,5)$
c. $(1,5]$
d. $[1,5]$

11. Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{n!}{(2n)!}(x-3)^n$.

a. 0
b. 1
c. 2
d. 4
e. ∞ .

12. Compute $\lim_{n\to\infty} \frac{(-1)^n 4n^3 + n}{(-1)^n 2n^3 + 3n}$.

a. $\frac{1}{3}$

b. $\frac{4}{5}$

c. 2

d. ∞

e. divergent but not to $\pm\infty$

13. The series $\sum_{n=1}^{\infty} \frac{3n^2}{n^3 + 2}$

a. converges by Simple Comparison with $\sum_{n=1}^{\infty} \frac{3}{n}$.

b. diverges by Simple Comparison with $\sum_{n=1}^{\infty} \frac{3}{n}$.

c. converges by the Integral Test.

d. diverges by the Integral Test.

e. diverges by the n^{th} Term Divergence Test.

14. If the series $S = \sum_{n=1}^{\infty} \frac{2n}{(n^2+2)^2}$ is approximated by its 100^{th} partial sum $S_{100} = \sum_{n=1}^{100} \frac{2n}{(n^2+2)^2}$

find a bound on the error $E_{100} = \sum_{n=101}^{\infty} \frac{2n}{(n^2+2)^2}$.

a. $|E_{100}| < \frac{2 \cdot 100}{(100^2+2)^2}$

b. $|E_{100}| < \frac{2 \cdot 101}{(101^2+2)^2}$

c. $|E_{100}| < \frac{1}{99^2+2}$

d. $|E_{100}| < \frac{1}{100^2+2}$

e. $|E_{100}| < \frac{1}{101^2+2}$

15. Compute $\lim_{x\to 0} \frac{\sin(x^3) - x^3}{x^9}$.

a. $-\frac{1}{3}$
b. $-\frac{1}{6}$
c. 0
d. $\frac{1}{6}$
e. ∞

Work Out: (Points indicated. Part credit possible. Show all work.)

16. (15 points) Compute $\int \frac{2}{x^3 - x}\, dx$.

a. Find the general partial fraction expansion. (Do not find the coefficients.)

$\frac{2}{x^3 - x} =$ ______________________________

b. Find the coefficients and plug them back into the expansion.

$\frac{2}{x^3 - x} =$ ______________________________

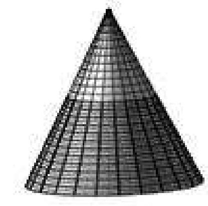
c. Compute the integral.

$\int \frac{2}{x^3 - x}\, dx =$ ______________________________

17. (15 points) A water tank has the shape of a cone with the vertex at the top. Its height is $H = 20$ ft and its radius is $R = 10$ ft.

It is filled with salt water to a depth of 10 ft which weighs $\delta = 64\ \frac{\text{lb}}{\text{ft}^3}$.

Find the work done to pump the water out the top of the tank.



18. (15 points) Consider the function $f(x) = \frac{1}{x^2}$.

a. Find the 3^{rd} degree Taylor polynomial for $f(x)$ centered at $x = 2$ by taking derivatives.

$T_3 f(x) =$ ____

b. Find the general term of its Taylor series and write the series in summation notation.

$Tf(x) =$ ____

c. Find the radius of convergence.