Name____________________________________

1-14	/49	16	/20
15	/15	17	/20
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MATH 172 Exam 1 Spring 2021

Sections 501 Solutions P. Yasskin

Multiple Choice: (Points indicated.)

1. (5 pts) Compute $\int_0^{\pi/4} x\cos(4x)\,dx$.

a. $-\frac{1}{8}$ correct choice

b. $-\frac{1}{16}$

c. 0

d. $\frac{1}{16}$

e. $\frac{1}{8}$

Solution: We use integration by parts with $\begin{array}{ll} u = x & dv = \cos(4x)\,dx \\ du = dx & v = \frac{1}{4}\sin(4x) \end{array}$. Then:

$$\int_0^{\pi/4} x\cos(4x)\,dx = \frac{x}{4}\sin(4x) - \frac{1}{4}\int \sin(4x)\,dx = \left[\frac{x}{4}\sin(4x) + \frac{1}{16}\cos(4x)\right]_0^{\pi/4}$$
$$= \left[\frac{\pi}{16}\sin(\pi) + \frac{1}{16}\cos(\pi)\right] - \left[\frac{0}{4}\sin(0) + \frac{1}{16}\cos(0)\right] = -\frac{1}{16} - \frac{1}{16} = -\frac{1}{8}$$

2. (5 pts) Compute $\int_0^{\pi/4} \tan^{5/2}x\sec^2x\,dx$.

a. $\frac{2}{3}$

b. $\frac{2}{5}$

c. $\frac{2}{7}$ correct choice

d. $\frac{2}{5}(2^{5/2}-1)$

e. $\frac{2}{7}(2^{5/2}-1)$

Solution: $u = \tan x \qquad du = \sec^2x\,dx$. $\quad \int_0^{\pi/4} \tan^{5/2}x\sec^2x\,dx = \int_0^1 u^{5/2}\,du = \left[\frac{2u^{7/2}}{7}\right]_0^1 = \frac{2}{7}$

3. (1 pts) In computing the integral $\int_0^{\pi/4} \tan^{5/2}x\sec^2x\,dx$, you used the formula (identity):

a. $\frac{d}{dx}\tan x = \sec^2 x$ correct choice

b. $\frac{d}{dx}\sec x = \sec x\tan x$

c. $\tan^2 x + 1 = \sec^2 x$

d. $\int \tan x\,dx = -\ln|\cos x| + C$

e. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

4. (5 pts) Compute $\int \frac{3x^5}{\sqrt{x^3+8}}\,dx$.

a. $\frac{2(x^3+8)^{3/2}}{3} - 16(x^3+8)^{1/2} + C$ correct choice

b. $\frac{2(x^3+8)^{5/2}}{5} - \frac{16(x^3+8)^{3/2}}{3} + C$

c. $\frac{2(x^3+8)^{3/2}}{3} + 16(x^3+8)^{1/2} + C$

d. $\frac{2(x^3+8)^{5/2}}{5} + \frac{16(x^3+8)^{3/2}}{3} + C$

e. $\frac{2u^{3/2}}{3} + 16u^{3/2} + C$

f. $\frac{2u^{5/2}}{5} + \frac{16u^{3/2}}{3} + C$

g. $\frac{2u^{3/2}}{3} + 16u^{3/2} + C$

h. $\frac{2u^{3/2}}{3} + 16u^{3/2} + C$

Solution: We make the substitution $u = x^3 + 8$. The $du = 3x^2\,dx$ and $x^3 = u - 8$. So:

$$\int \frac{3x^5}{\sqrt{x^3+8}}\,dx = \int \frac{u-8}{\sqrt{u}}\,du = \int (u^{1/2} - 8u^{-1/2})\,du = \frac{2u^{3/2}}{3} - 16u^{1/2} + C = \frac{2(x^3+8)^{3/2}}{3} - 16(x^3+8)^{1/2} + C$$

5. (5 pts) Compute $\int x^2 \ln|x|\,dx$.

a. $\frac{x^3}{3}\ln|x| + \frac{x^4}{12} + C$

b. $\frac{x^3}{3}\ln|x| - \frac{x^4}{12} + C$

c. $\frac{x^3}{3}\ln|x| + \frac{x^2}{6} + C$

d. $\frac{x^3}{3}\ln|x| - \frac{x^2}{6} + C$

e. $\frac{x^3}{3}\ln|x| + \frac{x^3}{9} + C$

f. $\frac{x^3}{3}\ln|x| - \frac{x^3}{9} + C$ correct choice

Solution: We use integration by parts with $\begin{array}{ll} u = \ln|x| & dv = x^2\,dx \\ du = \frac{1}{x}\,dx & v = \frac{x^3}{3} \end{array}$. Then:

$$\int x^2 \ln|x|\,dx = \frac{x^3}{3}\ln|x| - \frac{1}{3}\int x^3 \frac{1}{x}\,dx = \frac{x^3}{3}\ln|x| - \frac{x^3}{9} + C$$

6. (1 pts) In computing the integral $\int x^2 \ln|x|\,dx$, you used:

a. the substitution $u = x^2$

b. the substitution $u = \ln|x|$

c. integration by parts with $u = x^2$

d. integration by parts with $u = \ln|x|$ correct choice

e. $\int \ln x\,dx = x\ln x - x + C$

f. $\int \ln x\,dx = x\ln x + x + C$

7. (5 pts) Let $A(x)$ be the area under the graph of the function $y = f(x)$ above the x-axis between $x = 2$ and a variable point x. If $A(x) = x^4 - 16$, then $f(x) =$

a. 0

b. $\frac{x^5}{5} - 16x + \frac{128}{5}$

c. $4x^3 - 32$

d. $\frac{x^5}{5} - 16x$

e. $4x^3$ correct choice

Solution: $A(x) = \int_2^x f(t)\,dt \qquad \frac{dA}{dx} = f(x) \qquad f(x) = \frac{dA}{dx} = 4x^3$

8. (5 pts) Compute $\int_0^1 \frac{1}{\sqrt{16x^2+9}}\,dx$

a. $\frac{1}{4}\ln 6$

b. $\frac{1}{2}\ln 6$

c. $\frac{1}{4}\ln 3$ correct choice

d. $\frac{1}{2}\ln 3$

e. $2\ln 3$

Solution: Let $4x = 3\tan\theta$. Then $dx = \frac{3}{4}\sec^2\theta\,d\theta$ and

$$\int_0^1 \frac{1}{\sqrt{16x^2+9}}\,dx = \int_{x=0}^1 \frac{1}{\sqrt{9\tan^2\theta+9}}\frac{3}{4}\sec^2\theta\,d\theta = \frac{1}{4}\int_{x=0}^1 \frac{1}{\sqrt{\tan^2\theta+1}}\sec^2\theta\,d\theta$$

$$= \frac{1}{4}\int_{x=0}^1 \sec\theta\,d\theta = \frac{1}{4}\Big[\ln|\sec\theta+\tan\theta|\Big]_{x=0}^1 = \frac{1}{4}\left[\ln\left|\frac{\sqrt{16x^2+9}}{3} + \frac{4x}{3}\right|\right]_0^1$$

$$= \frac{1}{4}\ln\left|\frac{\sqrt{25}}{3} + \frac{4}{3}\right| - \frac{1}{4}\ln\left|\frac{\sqrt{9}}{3}\right| = \frac{1}{4}\ln\left|\frac{9}{3}\right| - \frac{1}{4}\ln 1 = \frac{1}{4}\ln 3$$

9. (1 pts) In computing the integral $\int_0^1 \frac{1}{\sqrt{16x^2+9}}\,dx$, you used:

a. the substitution $x = \frac{4}{3}\tan\theta$

b. the substitution $x = \frac{4}{3}\sin\theta$

c. the substitution $x = \frac{4}{3}\sec\theta$

d. the substitution $x = \frac{3}{4}\tan\theta$ correct choice

e. the substitution $x = \frac{3}{4}\sin\theta$

f. the substitution $x = \frac{3}{4}\sec\theta$

10. (1 pts) In computing the integral $\int_0^1 \frac{1}{\sqrt{16x^2+9}}\,dx$, you used the formula:

a. $\int \tan x\,dx = -\ln|\cos x| + C$

b. $\int \tan x\,dx = -\ln|\sin x| + C$

c. $\int \sec x\,dx = \ln|\sec x - \tan x| + C$

d. $\int \tan x\,dx = \ln|\cos x| + C$

e. $\int \tan x\,dx = \ln|\sin x| + C$

f. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$ correct choice

11. (5 pts) Find the length of the parametric curve $x = t^2$ and $y = \frac{1}{3}t^3 - t$ for $0 \le t \le 3$.

a. 3
b. 6
c. 9
d. 12 correct choice
e. 16

Solution: $\frac{dx}{dt} = 2t \qquad \frac{dy}{dt} = t^2 - 1$

$$L = \int_0^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_0^3 \sqrt{(2t)^2 + (t^2-1)^2}\,dt = \int_0^3 \sqrt{4t^2 + (t^4 - 2t^2 + 1)}\,dt$$

$$= \int_0^3 \sqrt{t^4 + 2t^2 + 1}\,dt = \int_0^3 \sqrt{(t^2+1)^2}\,dt = \int_0^3 (t^2+1)\,dt = \left[\frac{1}{3}t^3 + t\right]_0^3 = 9 + 3 = 12$$

12. (5 pts) The curve $y = \frac{e^x}{2} + \frac{1}{2e^x}$ for $0 \le x \le 1$ is rotated about the x-axis. Find the surface area swept out.

a. $\pi\left(\frac{e^2}{4} + \frac{3}{2} + \frac{1}{4e^2}\right)$
b. $\pi\left(\frac{e^2}{4} + \frac{1}{2} + \frac{1}{4e^2}\right)$
c. $\pi\left(\frac{e^2}{4} + 1 + \frac{1}{4e^2}\right)$
d. $\pi\left(\frac{e^2}{4} + \frac{3}{2} - \frac{1}{4e^2}\right)$
e. $\pi\left(\frac{e^2}{4} + \frac{1}{2} - \frac{1}{4e^2}\right)$
f. $\pi\left(\frac{e^2}{4} + 1 - \frac{1}{4e^2}\right)$ correct choice

Solution: $\frac{dy}{dx} = \frac{e^x}{2} - \frac{1}{2e^x}$. The radius is $r = y = \frac{e^x}{2} + \frac{1}{2e^x}$. So the surface area is:

$$A = \int_0^1 2\pi r\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx = \int_0^1 2\pi\left(\frac{e^x}{2} + \frac{1}{2e^x}\right)\sqrt{1 + \left(\frac{e^x}{2} - \frac{1}{2e^x}\right)^2}\,dx$$

$$= \int_0^1 2\pi\left(\frac{e^x}{2} + \frac{1}{2e^x}\right)\sqrt{\left(\frac{e^x}{2} + \frac{1}{2e^x}\right)^2}\,dx = \int_0^1 2\pi\left(\frac{e^x}{2} + \frac{1}{2e^x}\right)^2 dx$$

$$= \int_0^1 2\pi\left(\frac{e^{2x}}{4} + \frac{1}{2} + \frac{1}{4e^{2x}}\right)dx = 2\pi\left[\frac{e^{2x}}{8} + \frac{1}{2}x - \frac{1}{8e^{2x}}\right]_0^1 = \pi\left(\frac{e^2}{4} + 1 - \frac{1}{4e^2}\right) - \pi\left(\frac{1}{4} - \frac{1}{4}\right)$$

$$= \pi\left(\frac{e^2}{4} + 1 - \frac{1}{4e^2}\right)$$

13. (5 pts) A rocket takes off from rest $(v(0) = 0)$ at 10 meters above the ground (i.e. $y(0) = 10$) and has acceleration $a(t) = 6t + \sin t$. Find its height at $t = \pi$.

a. $\pi^3 + \pi + 9$
b. $\pi^3 + \pi + 10$ correct choice
c. $\pi^3 + \pi + 11$
d. $\pi^3 - \pi + 9$
e. $\pi^3 - \pi + 10$
f. $\pi^3 - \pi + 11$

Solution: $\frac{dv}{dt} = a = 6t + \sin t \qquad v = 3t^2 - \cos t + C \qquad v(0) = -1 + C = 0 \quad \Rightarrow \quad C = 1$

$\frac{dy}{dt} = v = 3t^2 - \cos t + 1 \qquad y = t^3 - \sin t + t + K \qquad y(0) = K = 10$

$y = t^3 - \sin t + t + 10 \qquad y(\pi) = \pi^3 + \pi + 10$

Work Out: (Points indicated. Part credit possible. Show all work.)

14. (15 pts) A bar between $x = 2$ and $x = 4$ has linear density $\delta = \dfrac{1}{x^3}$ g/cm.

a. Find the total mass of the bar.

Solution: $M = \int \delta\, dx = \int_2^4 \frac{1}{x^3}\, dx = \left[-\frac{1}{2x^2}\right]_2^4 = -\frac{1}{32} + \frac{1}{8} = \frac{3}{32}$

b. Find the center of mass of the bar.

Solution: $M_1 = \int x\delta\, dx = \int_2^4 \frac{x}{x^3}\, dx = \int_2^4 \frac{1}{x^2}\, dx = \left[-\frac{1}{x}\right]_2^4 = -\frac{1}{4} + \frac{1}{2} = \frac{1}{4}$

$\bar{x} = \frac{M_1}{M} = \frac{1}{4}\frac{32}{3} = \frac{8}{3}$

15. (20 pts) Compute $\int e^{2x} \sin 3x\, dx$.

The answer has the form $Ae^{2x} \sin 3x + Be^{2x} \cos 3x + C$. Then $A =$ ____ and $B =$ ____.

Solution: Use parts with $\begin{array}{ll} u = \sin 3x & dv = e^{2x}\, dx \\ du = 3\cos 3x\, dx & v = \frac{1}{2}e^{2x} \end{array}$. Then

$$I = \int e^{2x} \sin 3x\, dx = \frac{1}{2}e^{2x} \sin 3x - \frac{3}{2}\int e^{2x} \cos 3x\, dx$$

Next use parts with $\begin{array}{ll} u = \cos 3x & dv = e^{2x}\, dx \\ du = -3\sin 3x\, dx & v = \frac{1}{2}e^{2x} \end{array}$

$$I = \frac{1}{2}e^{2x} \sin 3x - \frac{3}{2}\left[\frac{1}{2}e^{2x} \cos 3x + \frac{3}{2}\int e^{2x} \sin 3x\, dx\right] = \frac{1}{2}e^{2x} \sin 3x - \frac{3}{4}e^{2x} \cos 3x - \frac{9}{4}I$$

$$I + \frac{9}{4}I = \frac{1}{2}e^{2x} \sin 3x - \frac{3}{4}e^{2x} \cos 3x$$

$$I = \frac{4}{13}\left(\frac{1}{2}e^{2x} \sin 3x - \frac{3}{4}e^{2x} \cos 3x\right) + C = \frac{2}{13}e^{2x} \sin 3x - \frac{3}{13}e^{2x} \cos 3x + C$$

16. (20 pts) Compute $\int \frac{\sqrt{x^2 - 9}}{x}\, dx$.

The answer has the form $A(x^2 - 9)^{3/2} + B\sqrt{x^2 - 9} + C \operatorname{arcsec} \frac{x}{3} + K$.

Then $A =$ ____, $B =$ ____ and $C =$ ____.

Solution: Let $x = 3\sec\theta$. Then $dx = 3\sec\theta\tan\theta\, d\theta$. So

$$I = \int \frac{\sqrt{x^2 - 9}}{x}\, dx = \int \frac{\sqrt{9\sec^2\theta - 9}}{3\sec^2\theta} 3\sec\theta\tan\theta\, d\theta = 3\int \tan^2\theta\, d\theta = 3\int \sec^2\theta - 1\, d\theta$$

$$= 3\tan\theta - 3\theta + K$$

Draw a triangle with hypotenous x, adjacent side 3 and opposite side $\sqrt{x^2 - 9}$. Then

$$I = 3\frac{\sqrt{x^2 - 9}}{3} - 3 \operatorname{arcsec} \frac{x}{3} + K = \sqrt{x^2 - 9} - 3 \operatorname{arcsec} \frac{x}{3} + K$$