Name_____

MATH 172 Final Spring 2021

Sections 501 P. Yasskin

1-12	/60	14	/15
13	/15	15	/15
		Total	/105

Anything above 100 is extra credit.

Multiple Choice and Short Answer: (5 Points Each)

1. Compute $\int_1^e 3x^2 \ln x \, dx.$

a. $\frac{2}{3}e^3 - \frac{1}{3}$ d. $\frac{2}{3}e^3$ g. $\frac{2}{3}e^3 + \frac{1}{3}$

b. $\frac{4}{3}e^3 - \frac{1}{3}$ e. $\frac{4}{3}e^3$ h. $\frac{4}{3}e^3 + \frac{1}{3}$

c. $2e^3 - \frac{1}{3}$ f. $2e^3$ i. $2e^3 + 1$

2. Compute $\int_0^{\pi/4} \sin^2 x \cos^2 x \, dx.$

a. $\frac{\pi}{32}$

b. $\frac{\pi}{16}$

c. $\frac{\pi}{8}$

d. $\frac{\pi}{4}$

e. $\frac{\pi}{2}$

3. Compute $\int \frac{1}{(x^2+4)^{3/2}} \, dx$

a. $\frac{\sqrt{x^2-4}}{4x} + C$ c. $\frac{1}{2}\arctan\frac{x}{2} + \frac{x}{4\sqrt{x^2+4}} + C$ e. $\frac{x}{2}\arctan\frac{x}{2} + C$

b. $\frac{x}{4\sqrt{x^2+4}} + C$ d. $\frac{1}{4}\arctan\frac{x}{4} + \frac{\sqrt{x^2-4}}{4x} + C$ f. $\frac{x}{4}\arctan\frac{x}{2} + C$

4. Find the area between the line $y = x$ and the parabola $x = 5y - y^2$.

a. 36
b. $\frac{80}{3}$
c. $\frac{32}{3}$
d. 18
e. $\frac{25}{3}$

5. Find the average value of the function $f(x) = 6x - x^2$ on $[0, 6]$.

a. 180
b. 36
c. 30
d. 6
e. $\frac{9}{2}$

6. Find the center of mass of a 2 m bar whose density is $\delta = \frac{1}{x^3}$ for $2 \leq x \leq 4$.

a. $\frac{7}{3}$
b. $\frac{1}{4}$
c. $\frac{3}{8}$
d. $\frac{8}{3}$
e. $\frac{5}{2}$

7. Find the arc length of the parametric curve $\vec{r}(t) = \left(\frac{1}{2}t^2, \frac{1}{3}t^3\right)$ for $0 \le t \le \sqrt{3}$.

a. 3

b. $\frac{8}{3}$

c. $\frac{7}{3}$

d. 2

e. $\frac{4}{3}$

8. The region between the parabola $x = 6y - y^2$ and the y-axis is rotated about the x-axis. Find the volume swept out.

a. $V = 2 \cdot 6^2\pi$

b. $V = 3 \cdot 6^2\pi$

c. $V = 6^3\pi$

d. $V = \frac{6^4}{5}\pi$

e. $V = 5 \cdot 6^4\pi$

9. Find the area inside the spiral $r = e^{\theta}$ for $0 \le \theta \le \pi$.

a. $\frac{1}{4}(e^{2\pi} - 1)$	i. $\frac{1}{4}e^{2\pi}$	q. $\frac{1}{4}(e^{2\pi} + 1)$
b. $\frac{1}{2}(e^{2\pi} - 1)$	j. $\frac{1}{2}e^{2\pi}$	r. $\frac{1}{2}(e^{2\pi} + 1)$
c. $e^{2\pi} - 1$	k. $e^{2\pi}$	s. $e^{2\pi} + 1$
d. $2(e^{2\pi} - 1)$	l. $2e^{2\pi}$	t. $2(e^{2\pi} + 1)$
e. $\frac{1}{4}(e^{\pi} - 1)$	m. $\frac{1}{4}e^{\pi}$	u. $\frac{1}{4}(e^{\pi} + 1)$
f. $\frac{1}{2}(e^{\pi} - 1)$	n. $\frac{1}{2}e^{\pi}$	v. $\frac{1}{2}(e^{\pi} + 1)$
g. $e^{\pi} - 1$	o. e^{π}	w. $e^{\pi} + 1$
h. $2(e^{\pi} - 1)$	p. $2e^{\pi}$	x. $2(e^{\pi} + 1)$

10. Find the arc length of the spiral $r = e^{\theta}$ for $0 \le \theta \le \pi$.

a. $e^{2\pi} + 1$
b. $\sqrt{2}\,(e^{2\pi} + 1)$
c. $2(e^{2\pi} + 1)$
d. $e^{\pi} + 1$
e. $\sqrt{2}\,(e^{\pi} + 1)$
f. $2(e^{\pi} + 1)$
g. $e^{2\pi}$
h. $\sqrt{2}\,e^{2\pi}$
i. $2e^{2\pi}$
j. e^{π}
k. $\sqrt{2}\,e^{\pi}$
l. $2e^{\pi}$
m. $e^{2\pi} - 1$
n. $\sqrt{2}\,(e^{2\pi} - 1)$
k. $2(e^{2\pi} - 1)$
o. $e^{\pi} + 1$
p. $\sqrt{2}\,(e^{\pi} - 1)$
q. $2(e^{\pi} - 1)$

11. Find the Taylor series for $f(x) = \frac{1}{x}$ about $x = 2$.

a. $\sum_{n=0}^{\infty} \frac{1}{2^n} x^n$
b. $\sum_{n=0}^{\infty} \frac{1}{2^n} (x-2)^n$
c. $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} x^n$
d. $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} (x-2)^n$
e. $\sum_{n=0}^{\infty} \frac{n!}{2^n} x^n$
f. $\sum_{n=0}^{\infty} \frac{n!}{2^n} (x-2)^n$
g. $\sum_{n=0}^{\infty} \frac{(-1)^n n!}{2^n} x^n$
h. $\sum_{n=0}^{\infty} \frac{(-1)^n n!}{2^n} (x-2)^n$
i. $\sum_{n=0}^{\infty} \frac{1}{2^{n+1}} x^n$
j. $\sum_{n=0}^{\infty} \frac{1}{2^{n+1}} (x-2)^n$
k. $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} x^n$
l. $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} (x-2)^n$

12. Compute $\lim_{n\to\infty} n^2 \left[1 - \cos\left(\frac{1}{n}\right) \right]$.

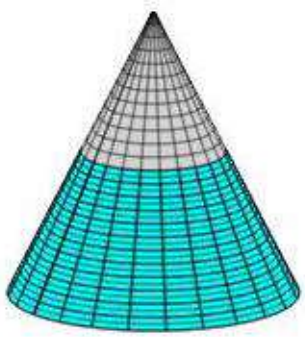
a. -1
b. $-\frac{1}{2}$
c. $-\frac{1}{4}$
d. 0
e. $\frac{1}{4}$
f. $\frac{1}{2}$
g. 1
h. π
i. 2π

Work Out: (Points indicated. Part credit possible. Show all work.)

13. (15 points) Work Out Problem

A water tank has the shape of a cone with the vertex at the top. Its height is $H = 16$ ft and its radius is $R = 8$ ft. It is filled with salt water to a depth of 10 ft which weighs $\delta = 64 \frac{\text{lb}}{\text{ft}^3}$.

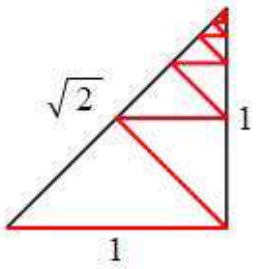
Find the work done to pump the water out the top of the tank.



$W =$ _____

14. (15 points) Work Out Problem

Find the length of the infinite zigzag within the $45°$ right triangle, shown at the right. Each diagonal is at $45°$. The total length includes the base.



$L =$ _____

15. (15 points) Work Out Problem

Find the interval of convergence of the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n}+1} \frac{(x-4)^n}{2^n}$.

a. Find the radius of convergence.

$R =$ _____

b. Check the convergence at the left endpoint.
Be sure to name any convergence test you use and check out all conditions.

☐ Converges ☐ Diverges

c. Check the convergence at the right endpoint.
Be sure to name any convergence test you use and check out all conditions.

☐ Converges ☐ Diverges

d. State the interval of convergence.

Interval = _____