Name_____

MATH 172 Final Exam Spring 2023

Sections 502 P. Yasskin

1-10	/50	13	/18
12	/18	14	/20
		Total	/106

Multiple Choice: (5 points each. No part credit. Circle your answers.)

1. $\int_0^\pi \sin^3 x\,dx =$

a. $\frac{1}{3}$

b. $\frac{2}{3}$

c. $\frac{4}{3}$

d. $\frac{3}{8}\pi$

e. $\frac{3}{4}\pi$

2. $\int \frac{1}{x^2\sqrt{4x^2-9}}\,dx =$

a. $\frac{\sqrt{4x^2-9}}{9x} + C$

b. $\frac{9x}{\sqrt{4x^2-9}} + C$

c. $\frac{2\sqrt{4x^2-9}}{27} + C$

d. $\frac{4}{27}\ln\left(\frac{2x}{3} + \frac{\sqrt{4x^2-9}}{3}\right) + C$

e. $\frac{4}{27}\ln\frac{\sqrt{4x^2-9}}{2x} - \frac{1}{27}\frac{4x^2-9}{2x^2}$

3. In the partial fraction expansion, $\frac{8}{x^3+4x} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$, which coefficient is **right**?

a. $A = 1$
b. $B = -2$
c. $B = 2$
d. $C = -2$
e. $C = 2$

4. Approximate $\int_2^{14} \frac{144}{x^2}\,dx$ using a midpoint Riemann sum with 3 intervals.

a. $\frac{49}{4}$
b. $\frac{74}{3}$
c. 62
d. 74
e. 49

5. Find the arc length of the curve $(x,y,z) = \left(t, t^2, \frac{2}{3}t^3\right)$ between $t = 0$ and $t = 1$.

a. $\frac{5}{3}$
b. $\frac{8}{3}$
c. $\frac{16}{3}$
d. 2
e. 4

6. The curve $y = x^2$ between $x = 0$ and $x = \sqrt{2}$ is revolved about the y-axis. Find the area of the surface swept out.

a. 3π
b. $\frac{7}{4}\pi$
c. $\frac{9}{2}\pi$
d. 4π
e. $\frac{13}{3}\pi$

7. A sequence is defined recursively by $a_1 = 3$ and $a_{n+1} = \frac{{a_n}^2 + 7}{8}$. Find $\lim_{n\to\infty} a_n$.

a. 0
b. 1
c. 2
d. 3
e. 7

8. The series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1+\sqrt{n}}{n^2+\sqrt{n}}$ is:

a. Absolutely Convergent
b. Conditionally Convergent
c. Divergent
d. Conditionally Divergent

9. The series $\sum_{n=1}^{\infty} \frac{1+n}{n+n^4}$ is:

a. convergent by Simple Comparison with $\sum_{n=1}^{\infty} \frac{1}{n^3}$

b. convergent by Limit Comparison but not Simple Comparison with $\sum_{n=1}^{\infty} \frac{1}{n^3}$

c. divergent by Simple Comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$

d. divergent by Limit Comparison but not Simple Comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$

10. $\lim_{x \to 0} \frac{\sin(x^3) - x^3}{x^9} =$

a. ∞

b. $\frac{1}{6}$

c. 0

d. $-\frac{1}{6}$

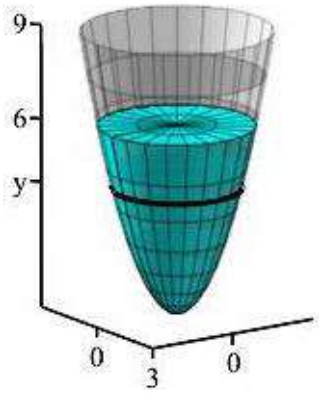
e. $-\infty$

Work Out: (Points indicated. Part credit possible. Show all work.)

11. (18 points) The area below $y = e^{-x}$ between $x = 0$ and $x = 2$ is revolved about the y-axis. Find the volume of the solid swept out.

12. (18 points) The curve $y = x^2$ for $y \le 9$ is revolved about the y-axis to form a bowl. It is filled to a depth of $y = 6$ with salt water with weight density $g\delta = 64 \frac{\text{lb}}{\text{ft}^3}$.

How much work is done to pump the water out the top of the bowl.



13. (20 points) Find the interval of convergence of the series $\sum_{n=2}^{\infty} \frac{2^n + 4}{6^n + 12}(x-5)^n$ as follows:

a. Find the radius of convergence.

b. Check convergence at the right endpoint.

c. Check convergence at the left endpoint.

d. State the interval of convergrnce.