Name_____ ID_____

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MATH 172 EXAM 3 Fall 1998

Section 502 P. Yasskin

Multiple Choice: (5 points each)

1. Compute $\lim_{n\to\infty} \frac{3n^2}{1+n^3}$

a. 0
b. 1
c. 2
d. 3
e. Divergent

2. Find r such that $5 + 5r + 5r^2 + 5r^3 + 5r^4 + \cdots = 3.$

a. $\frac{2}{5}$
b. $-\frac{2}{5}$
c. $\frac{3}{5}$
d. $\frac{5}{3}$
e. $-\frac{2}{3}$

3. The series $\sum_{n=1}^{\infty} \frac{1}{n^2+\sqrt{n}}$ is

a. divergent by comparison to $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.
b. convergent by comparison to $\sum_{n=1}^{\infty} \frac{1}{n^2}$.
c. divergent by the ratio test.
d. convergent by the ratio test.
e. divergent by the n^{th}-term test.

4. Compute $\displaystyle\sum_{k=1}^{99}\left(\frac{1}{\sqrt{k}}-\frac{1}{\sqrt{k+1}}\right)$

a. .9
b. .99
c. 1
d. 1.1
e. Divergent

5. Compute $\displaystyle\sum_{n=1}^{\infty}\frac{3n^2}{1+n^3}$

a. $\ln 2$
b. $\frac{3}{2}$
c. $\frac{27}{82}$
d. Convergent but none of the above
e. Divergent

6. The series $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{3\sqrt{n}}$ is

a. absolutely convergent.
b. conditionally convergent.
c. absolutely divergent.
d. conditionally divergent.
e. oscillatory divergent.

7. Compute $\displaystyle\lim_{x\to 0}\frac{\cos(2x)-1+2x^2}{x^4}$

a. 0
b. $\frac{1}{24}$
c. $\frac{1}{12}$
d. $\frac{2}{3}$
e. ∞

8. Given that $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ (for $|x| < 1$), then (for $|x| < 1$) we have $\sum_{n=0}^{\infty} n x^n =$

a. $\frac{1}{1-x}$
b. $\frac{1}{(1-x)^2}$
c. $\frac{x}{(1-x)^2}$
d. $\frac{x}{1-x}$
e. $\frac{n}{1-x}$

9. The series $\sum_{n=0}^{\infty} \frac{1}{n!}\left(\frac{1}{2}\right)^n$ converges to

a. $\ln 2$
b. $\sqrt{e}$
c. $\sin\left(\frac{1}{2}\right)$
d. $\sin(2)$
e. e^2

10. Find the 3^{rd} degree term in the Taylor series for $f(x) = \frac{1}{x}$ centered at $x = 2$.

a. $\frac{3}{8}(x-2)^3$
b. $\frac{-3}{8}(x-2)^3$
c. $-6(x-2)^3$
d. $\frac{-1}{16}(x-2)^3$
e. $\frac{1}{16}(x-2)^3$

11. (15 points) Find the interval of convergence for the series $\sum_{n=0}^{\infty} \frac{(x-5)^n}{3^n n^3}$.

Be sure to identify each of the following and give reasons:

(1 pt) Center of Convergence: $a =$ _____

Radius of Convergence: $R =$ _____ (5 pt)

(1 pt) Right Endpoint: $x =$ _____

At the Right Endpoint the Series { Converges / Diverges } (circle one) (3 pt)

(1 pt) Left Endpoint: $x =$ _____

At the Left Endpoint the Series { Converges / Diverges } (circle one) (3 pt)

(1 pt) Interval of Convergence: _____

12. (15 points) Let $f(x) = x^2 \cos x$.

a. (10 pt) Find the Maclaurin series for $f(x)$. Write the series in summation form and also write out the first 4 terms.

b. (5 pt) Find $f^{(6)}(0)$.

13. (10 points) Given that $\ln(1+t) = t - \frac{1}{2}t^2 + \frac{1}{3}t^3 - \cdots,$ find the 6^{th} degree Taylor polynomial approximation about $x = 0$ for $\ln(1+x^2)$.

14. (15 points) You are given: $e^{(-x^2)} = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{n!} = 1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6} + \cdots.$

a. (10 pt) Use the quadratic Taylor polynomial approximation about $x = 0$ for $e^{(-x^2)}$ to estimate $\int_0^{0.1} e^{(-x^2)}\,dx$. (Keep 8 digits.)

b. (5 pt Extra Credit) Your result in (a) is equal to $\int_0^{0.1} e^{(-x^2)}\,dx$ to within $\pm$ how much? Why?