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MATH 172 FINAL EXAM Fall 1998

Section 502 P. Yasskin

Multiple Choice: (8 points each)

1. Compute $\int_0^{\sqrt{\pi}} x\sin(x^2)\ dx$

a. $-\frac{1}{2}$

b. 0

c. $\frac{1}{2}$

d. 1

e. 2

2. Compute $\int_0^1 x^2\ e^x\ dx$

a. $-3e$

b. $-3e+2$

c. $-3e-2$

d. e

e. $e-2$

3. Find the average value of the function $f(x) = 3x^2 + 1$ for $1 \le x \le 3$.

a. 13

b. 14

c. 15

d. 16

e. 17

4. Compute $\int \frac{x^2}{(1-x^2)^{3/2}}\,dx$

Hints: $\sin^2\theta + \cos^2\theta = 1$ $\tan^2\theta + 1 = \sec^2\theta$

a. $\frac{x}{\sqrt{1-x^2}} - \arctan x$

b. $\frac{x}{\sqrt{1-x^2}} + \arctan x$

c. $\frac{x}{\sqrt{1-x^2}} - \arcsin x$

d. $\frac{x}{\sqrt{1-x^2}} + \arcsin x$

e. $\frac{x}{\sqrt{1-x^2}} + x$

5. Find the area between the curves $y = x^2$ and $y = x^3$ for $0 \le x \le 1$.

a. $\frac{1}{24}$

b. $\frac{1}{12}$

c. $\frac{1}{7}$

d. $\frac{1}{6}$

e. 1

6. The area between the curves $y = x^2$ and $y = x^3$ for $0 \le x \le 1$ is rotated around the x-axis. Find the volume swept out.

a. $\frac{\pi}{5}$

b. $\frac{\pi}{10}$

c. $\frac{\pi}{20}$

d. $\frac{2\pi}{35}$

e. $\frac{4\pi}{35}$

7. Find the total mass of a 6 cm bar whose linear density is $\rho = (1 + x)$ g/cm where x is the distance from one end in cm.

a. $\frac{7}{6}$ g

b. 4 g

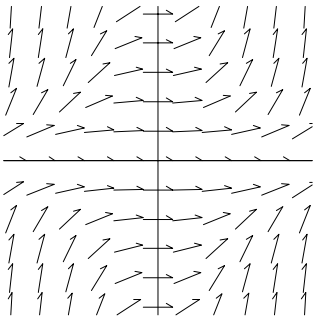
c. 6 g

d. 7 g

e. 24 g

8. Which of the following is the direction field for the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2}$?

a.



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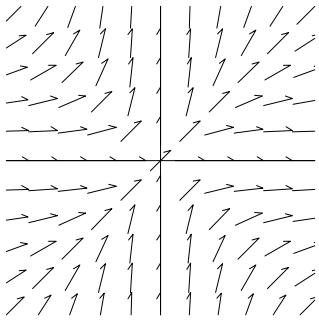

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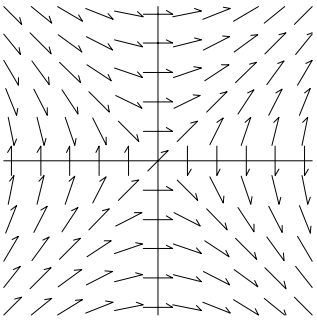

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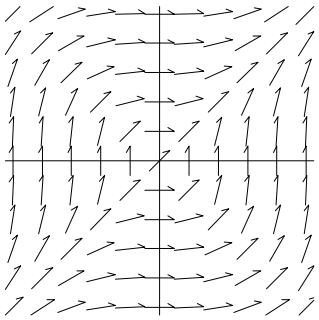
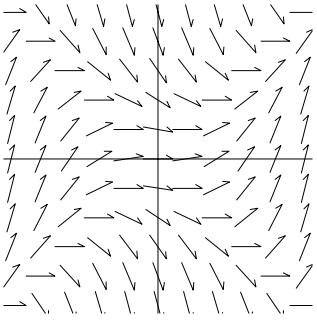


c.



9. Compute $\int_1^{\infty} \frac{1}{1+x^2}\, dx$

a. $\frac{\pi}{4}$

b. $\frac{\pi}{2}$

c. $\frac{3\pi}{4}$

d. Convergent but none of the above

e. Divergent

10. Compute $\sum_{n=1}^{\infty} \frac{1}{1+n^2} =$

a. $\frac{\pi}{4}$

b. $\frac{\pi}{2}$

c. $\frac{3\pi}{4}$

d. Convergent but none of the above

e. Divergent

11. The series $\sum_{n=1}^{\infty} (-1)^n \frac{n}{1+n^2}$ is

a. absolutely convergent

b. conditionally convergent

c. divergent

d. none of these

12. Find the radius of convergence of the series $\sum_{n=2}^{\infty} \frac{(x-3)^n}{2^n n^2}$.

a. 0
b. 1
c. 2
d. 3
e. 4

13. The vectors $\vec{a} = (2, -1, 4)$ and $\vec{b} = (3, 2, -1)$ are

a. parallel
b. perpendicular
c. neither

14. Find the area of the parallelogram whose edges are $\vec{a} = (1, 2, 3)$ and $\vec{b} = (3, 2, 1)$.

a. $2\sqrt{2}$
b. $4\sqrt{2}$
c. $4\sqrt{3}$
d. $2\sqrt{6}$
e. $4\sqrt{6}$

15. A baseball is thrown straight North initially at 45° above horizontal and follows a parabolic path, up and back down. At the top of the trajectory, in what direction does the unit binormal $\hat{B}$ point?

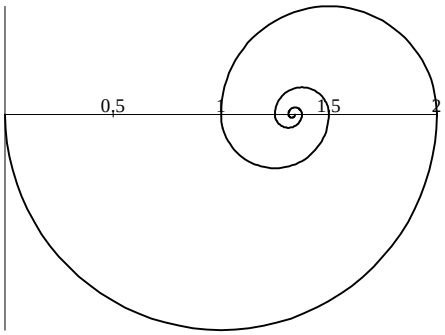
a. West and horizontal
b. West and below horizontal
c. Straight down
d. East and below horizontal
e. East and horizontal

Work-out Problems: (20 points each)

16. Solve the differential equation $\frac{dy}{dx} + 2xy = e^{-x^2}$ with the initial condition $y(1) = 0$.

17. Find the point where the line $\left\{ \begin{array}{l} x = -2 + t \\ y = 1 + 2t \\ z = 3 - 2t \end{array} \right\}$ intersects the plane $2x - 3y + z = -16$.

18. The spiral at the right is made from an infinite number of semicircles whose centers are all on the x-axis. The radius of each semicircle is half of the radius of the previous semicircle.



a. Consider the infinite sequence of points where the spiral crosses the x-axis. What is the x-coordinate of the limit of this sequence?

b. What is the total length of the spiral (with an infinite number of semicircles)? Or, is the length infinite?

19. Consider the twisted cubic curve $\vec{r}(t) = (6t, 3t^2, t^3)$ for $0 \leq t \leq 2$.

a. Find the arc length of the curve between $t = 0$ and $t = 2$.

b. Find the unit binormal vector $\hat{B}$.