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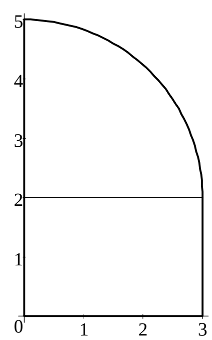
MATH 172 EXAM 1 Spring 1999

Section 504 Solutions P. Yasskin

Multiple Choice: (6 points each)

1. Evaluate $\int_0^3 2+\sqrt{9-x^2}\,dx$ by interpretating it as an area.

- **a.** $6+\frac{9\pi}{4}$ correctchoice
- **b.** $3+9\pi$
- **c.** $2+\frac{9\pi}{2}$
- **d.** $2+9\pi$
- **e.** $6+9\pi$



$$\int_0^3 2+\sqrt{9-x^2}\,dx = \text{rectangle} + \text{quarter circle}$$
$$= 3 \bullet 2 + \frac{1}{4}\pi(3)^2 = 6+\frac{9\pi}{4}$$

2. Compute: $\int_1^2 \left(x^3+x+\frac{1}{x^3}\right)dx$

- **a.** $\frac{47}{8}$
- **b.** $\frac{45}{8}$ correctchoice
- **c.** $\frac{351}{64}$
- **d.** $\frac{415}{64}$
- **e.** $\frac{417}{64}$

$$\int_1^2 \left(x^3+x+\frac{1}{x^3}\right)dx = \left[\frac{x^4}{4}+\frac{x^2}{2}-\frac{1}{2x^2}\right]_1^2 = \left[4+2-\frac{1}{8}\right]-\left[\frac{1}{4}+\frac{1}{2}-\frac{1}{2}\right] = \frac{45}{8}$$

3. Compute: $\int_0^{\pi/4} \cos^3\theta \sin\theta \; d\theta$

a. $-\frac{3}{16}$

b. $-\frac{1}{16}$

c. $\frac{1}{16}$

d. $\frac{3}{16}$ correctchoice

e. 3

$u = \cos\theta \qquad du = -\sin\theta \; d\theta$

$$\int_0^{\pi/4} \cos^3\theta \sin\theta \; d\theta = -\int_{\theta=0}^{\pi/4} u^3 \; du = -\left[\frac{u^4}{4} \right] = -\left[\frac{\cos^4\theta}{4} \right]_0^{\pi/4}$$

$$= -\left[\frac{1}{4}\left(\frac{1}{\sqrt{2}} \right)^4 \right] + \left[\frac{1}{4} \right] = -\frac{1}{16} + \frac{1}{4} = \frac{3}{16}$$

4. Compute: $\frac{d}{dx} \int_x^{x^2} e^{t^2} dt$

a. $e^{x^4} - e^{x^2}$

b. $e^{x^4} 2x - e^{x^2}$ correctchoice

c. $e^{x^4} 4x^3 - e^{x^2}$

d. $e^{4x^2} - e^{x^2}$

e. $e^{4x^2} 2x - e^{x^2}$

Let $F(t)$ be an antiderivative of e^{t^2}. Then $F'(t) = e^{t^2}$. So $\int_x^{x^2} e^{t^2} dt = F(x^2) - F(x)$.

Thus by chain rule: $\frac{d}{dx} \int_x^{x^2} e^{t^2} dt = F'(x^2) \bullet 2x - F'(x) = e^{x^4} 2x - e^{x^2}$

5. Compute: $\int \frac{x^2 + 1}{(x^3 + 3x)^2} dx$

a. $\frac{-3}{x^3 + 3x} + C$

b. $\frac{-1}{x^3 + 3x} + C$

c. $\frac{-1}{3x^3 + 9x} + C$ correctchoice

d. $\frac{1}{x^3 + 3x} + C$

e. $\frac{3}{x^3 + 3x} + C$

$u = x^3 + 3x \qquad du = (3x^2 + 3)\,dx = 3(x^2 + 1)\,dx \qquad \frac{1}{3} du = (x^2 + 1)\,dx$

$$\int \frac{x^2 + 1}{(x^3 + 3x)^2} dx = \frac{1}{3} \int \frac{1}{u^2} du = \frac{-1}{3u} + C = \frac{-1}{3x^3 + 9x} + C$$

6. Compute: $\int_1^e x^4 \ln x \; dx$

a. $\frac{4}{25}e^5 + \frac{1}{25}$ correctchoice

b. $\frac{4}{25}e^5 - \frac{1}{25}$

c. $\frac{4}{25}(e^5 - 1)$

d. $\frac{4}{25}(e^5 + 1)$

e. $\frac{1}{5}(e^5 + 1)$

$u = \ln x \qquad dv = x^4\,dx$

$du = \frac{1}{x}\,dx \quad v = \frac{x^5}{5}$

$$\int_1^e x^4 \ln x \; dx = \left[\frac{x^5}{5}\ln x - \int \frac{x^4}{5} \; dx \right]_1^e = \left[\frac{x^5}{5}\ln x - \frac{x^5}{25} \right]_1^e$$
$$= \left[\frac{e^5}{5}\ln e - \frac{e^5}{25} \right] - \left[\frac{1}{5}\ln 1 - \frac{1}{25} \right] = \left(\frac{1}{5} - \frac{1}{25} \right)e^5 + \frac{1}{25} = \frac{4}{25}e^5 + \frac{1}{25}$$

7. Find the area between the curves $y = 12 - x^2$ and $y = y = 2x^2$.

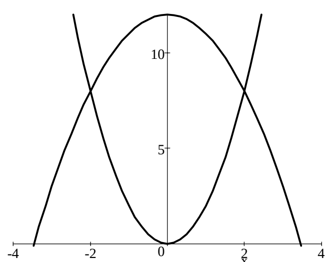
a. 2

b. 4

c. 12

d. 24

e. 32 correctchoice



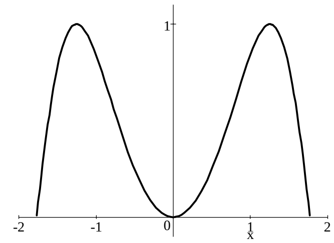
$12 - x^2 = 2x^2$

$12 = 3x^2$

$x = \pm 2$

$$A = \int_{-2}^{2} (12 - x^2) - (2x^2) \; dx = \int_{-2}^{2} (12 - 3x^2) \; dx = [12x - x^3]_{-2}^{2} = 32$$

8. The region below $y = \sin(x^2)$ above the x-axis for $0 \le x \le \sqrt{\pi}$ is rotated about the y-axis. Find the volume of the solid swept out.



a. $\frac{\pi}{2}$

b. π

c. 2π correctchoice

d. 3π

e. 4π

Thin cylinders with radius $r = x$ and height $h = \sin(x^2)$.

$$V = \int_0^{\sqrt{\pi}} 2\pi x \sin(x^2)\ dx = -\pi \cos(x^2)\Big|_0^{\sqrt{\pi}} = -\pi\cos\pi + \pi\cos 0 = 2\pi$$

9. The force needed to stretch a **superspring** x m beyond its natural length is $F = kx^3$. If it takes 32 N to stretch the superspring by 2 m, how much work is done in stretching it from 1 m to 2 m?

a. 3

b. 7

c. 15 correctchoice

d. 31

e. 63

$$F = kx^3 \qquad 32 = k\,2^3 = 8k \qquad k = 4 \qquad F = 4x^3$$

$$W = \int F dx = \int_1^2 4x^3\ dx = x^4\Big|_1^2 = 15$$

10. The mass density of a 9 m rod is $\rho = \frac{18}{(1+x)^3}\ \frac{\text{kg}}{\text{m}}$ for $0 \le x \le 9$. Find the average density of the rod.

a. $.495\ \frac{\text{kg}}{\text{m}}$

b. $.99\ \frac{\text{kg}}{\text{m}}$ correctchoice

c. $.4995\ \frac{\text{kg}}{\text{m}}$

d. $.999\ \frac{\text{kg}}{\text{m}}$

e. $.49995\ \frac{\text{kg}}{\text{m}}$

$$\rho_{\text{ave}} = \frac{1}{9}\int_0^9 \frac{18}{(1+x)^3}\,dx = 2\left[\frac{(1+x)^{-2}}{-2}\right]_0^9 = \left[\frac{-1}{(1+x)^2}\right]_0^9 = \frac{-1}{100} - \frac{-1}{1} = .99$$

11. (10 points) Compute: $\int_0^1 \frac{\arctan x}{1+x^2}\,dx$

$$\int_0^1 \frac{\arctan x}{1+x^2}\,dx = \int_0^{\pi/4} u\,du \qquad u = \arctan x$$

$$= \frac{u^2}{2}\Big|_0^{\pi/4} = \frac{\pi^2}{32} \qquad du = \frac{1}{1+x^2}\,dx$$

$$x = 1 \quad @ \quad u = \arctan 1 = \frac{\pi}{4}$$

OR

$$x = 0 \quad @ \quad u = \arctan 0 = 0$$

$$= \int_{x=0}^1 u\,du = \frac{u^2}{2}\Big|_{x=0}^1 = \frac{(\arctan x)^2}{2}\Big|_0^1 = \frac{(\arctan 1)^2}{2} - \frac{(\arctan 0)^2}{2} = \frac{\pi^2}{32}$$

12. (10 points) Compute: $\int \sec^3\theta\ \tan^3\theta\ d\theta$

$$\int \sec^3\theta \tan^3\theta\ d\theta = \int \sec^2\theta \tan^2\theta\ \sec\theta \tan\theta\ d\theta \qquad du = \sec\theta\tan\theta\ d\theta$$

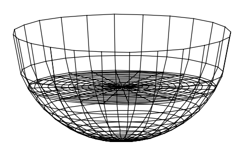
$$= \int u^2(u^2-1)\ du \qquad u = \sec\theta$$

$$= \int (u^4 - u^2)\ du \qquad \tan^2\theta = \sec^2\theta - 1 = u^2 - 1$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + C = \frac{\sec^5\theta}{5} - \frac{\sec^3\theta}{3} + C$$

13. (10 points)

A hemispherical bowl of radius 2 ft
is filled to a depth of 1 ft.
Find the volume of the water.
HINT: Slice it horizontally.



Put the zero of the vertical y-axis at the center of the sphere and the positive direction down. Then the water goes from $y = 1$ to $y = 2$. A slice perpendicular to the y-axis at height y is a disk of radius $r = \sqrt{4-y^2}$.

$$V = \int \pi r^2\ dy = \int_1^2 \pi\left(\sqrt{4-y^2}\right)^2 dy = \int_1^2 \pi(4-y^2)\ dy = \pi\left[4y - \frac{y^3}{3}\right]_1^2 = \frac{5\pi}{3}$$

14. (10 points) How much work is done in pumping the water out the top of the bowl shown in problem **13**? Leave the density as ρ and the acceleration of gravity as g.

The volume of the slice at height y is $dV = \pi\left(\sqrt{4-y^2}\right)^2 dy = \pi(4-y^2)\ dy$. So its weight is $dF = \rho g\ dV = \rho g\ \pi(4-y^2)\ dy$. This slice must be lifted a distance $D = y$. So the work done is $W = \int D\ dF = \int_1^2 y\ \rho g\ \pi(4-y^2)\ dy = \rho g\ \pi \int_1^2 (4y - y^3)\ dy = \rho g\ \pi\left[2y^2 - \frac{y^4}{4}\right]_1^2$

$$= \rho g\ \pi[8-4] - \rho g\ \pi\left[2 - \frac{1}{4}\right] = \frac{9\rho g\ \pi}{4}$$