Name_____ ID_____

1-14	/70
15	/10
16	/10
17	/10+4

MATH 172   FINAL EXAM   Spring 1999

Section 504   P. Yasskin

Multiple Choice: (5 points each)

1. If $\vec{a} = (2,0,1)$ and $\vec{b} = (-1,2,1)$ then $2\vec{a} - 3\vec{b} =$

   a. $(1,6,5)$
   b. $(1,-6,5)$
   c. $(7,6,-1)$
   d. $(7,-6,-1)$
   e. $(-1,6,-5)$

2. If $\vec{a} = (2,0,1)$ and $\vec{b} = (-1,2,1)$ then $\vec{a} \times \vec{b} =$

   a. $(-2,-3,4)$
   b. $(-2,3,4)$
   c. $(-2,0,1)$
   d. $(4,0,-1)$
   e. $(4,3,-2)$

3. If $\vec{a} = (2,0,1)$ and $\vec{b} = (-1,2,1)$ are two edges of a triangle, find the area of the triangle.

   a. $\sqrt{3}$
   b. $\dfrac{3}{2}$
   c. $\dfrac{1}{2}\sqrt{29}$
   d. $\dfrac{29}{2}$
   e. $\sqrt{29}$

1

4. If $\vec{u} = \left(\sqrt{2}, -1, 1\right)$ and $\vec{v} = (0, 1, -1)$ then the angle between $\vec{u}$ and $\vec{v}$ is $\theta =$

   a. $30°$
   b. $45°$
   c. $60°$
   d. $120°$
   e. $135°$

5. The series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} (x-2)^n$ is the Taylor series about $x = 2$ for

(Hint: Just sum the series.)

   a. $\dfrac{1}{x}$
   b. $\dfrac{2}{x}$
   c. $\dfrac{1}{x-2}$
   d. $\dfrac{2}{x-2}$
   e. $\dfrac{1}{2(x-2)}$

6. Find the radius of convergence of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} (x-2)^n$.

   a. 0
   b. 1
   c. 2
   d. 4
   e. 8

7. The series $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n!}$ is

    a. Divergent by the nth term Divergence Test

    b. Convergent by the Ratio Test

    c. Divergent by the Ratio Test

    d. Convergent by the Integral Test

    e. Divergent by the Integral Test

8. Compute $\displaystyle\lim_{x\to 0} \frac{\sin(2x) - 2x + \frac{8x^3}{3}}{x^5}$

    a. $-\infty$

    b. 0

    c. $\dfrac{4}{3}$

    d. $\dfrac{2}{5!}$

    e. ∞

9. Compute $\displaystyle\int (2x^3 - 1)\sin(x^4 - 2x)\ dx$

    a. $2\cos(x^4 - 2x) + C$

    b. $-2\cos(x^4 - 2x) + C$

    c. $\dfrac{1}{2}\cos(x^4 - 2x) + C$

    d. $-\dfrac{1}{2}\cos(x^4 - 2x) + C$

    e. $\cos(x^4 - 2x) + C$

10. Compute $\displaystyle\int_0^2 \frac{x^2}{\sqrt{4-x^2}}\, dx$

    a. 1

    b. 2

    c. $\dfrac{\pi}{4}$

    d. π

    e. 2π

11. Compute $\displaystyle\int_0^{\pi/2} \sin^6 x \cos^3 x\, dx$

    a. $\dfrac{1}{63}$

    b. $\dfrac{2}{63}$

    c. $\dfrac{1}{21}$

    d. $\dfrac{4}{63}$

    e. $\dfrac{2}{21}$

12. $\displaystyle\int_1^\infty \frac{1}{x+e^{2x}}\, dx$ is

    a. Convergent by comparison to $\displaystyle\int_1^\infty \frac{1}{e^{2x}}\, dx$

    b. Divergent by comparison to $\displaystyle\int_1^\infty \frac{1}{e^{2x}}\, dx$

    c. Convergent by comparison to $\displaystyle\int_1^\infty \frac{1}{x}\, dx$

    d. Divergent by comparison to $\displaystyle\int_1^\infty \frac{1}{x}\, dx$

13. The basket shown at the right is 9 in tall. Its horizontal cross sections are circles whose radius is given by $r = 2\sqrt{y}$ where y is the height from the bottom. Find the volume of the basket.
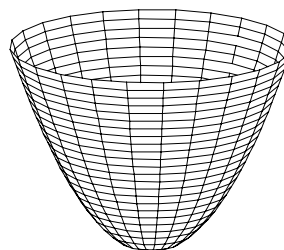
- **a**. 216π
- **b**. 162π
- **c**. 108π
- **d**. 81π
- **e**. $\dfrac{81\pi}{2}$

14. A trough filled with water is 3m long and its end is a $45°$ right triangle which is 2m high and 2m wide. Find the work done to pump the water out of the top.
(ρ is the density of water and g is the acceleration of gravity.)
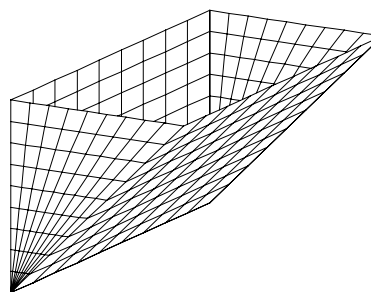
- **a**. $2\rho g$
- **b**. $3\rho g$
- **c**. $4\rho g$
- **d**. $6\rho g$
- **e**. $8\rho g$

15. (10 points) Compute $\displaystyle\int_0^1 x \arctan x \; dx.$

16. (10 points) Solve the differential equation $x^2 \dfrac{dy}{dx} + xy = \dfrac{2x^2}{1 + x^2}$ with the initial condition $y(1) = \ln 4.$

17. (10 points) Approximate $\displaystyle\int_0^{0.1} e^{-x^2}\,dx$ to 7 decimal places.

(4 points extra credit:) How do you know it is accurate to 7 decimal places?