MATH 172 EXAM 2 Fall 1999

Section 502 Solutions P. Yasskin

Multiple Choice: (9 points each)

1. $\int_0^2 \frac{x}{4-x^2}\,dx =$

- **a.** $-\infty$
- **b.** $-\ln 4$
- **c.** 0
- **d.** $\ln 4$
- **e.** ∞ correctchoice

$u = 4 - x^2 \qquad du = -2x\,dx \qquad -\frac{1}{2}\,du = x\,dx$

$$\int_0^2 \frac{x}{4-x^2}\,dx = -\frac{1}{2}\int \frac{1}{u}\,du = -\frac{1}{2}\ln|u| = \left[-\frac{1}{2}\ln|4-x^2|\right]_0^2$$

$$= \lim_{x\to 2^-}\left[-\frac{1}{2}\ln|4-x^2|\right] - \left[-\frac{1}{2}\ln|4|\right] = -\frac{1}{2}(-\infty) = \infty$$

2. $\int_2^\infty \frac{2+x}{x^3}\,dx =$

- **a.** $-\infty$
- **b.** $\frac{3}{4}$ correctchoice
- **c.** 1
- **d.** 2
- **e.** ∞

$$\int_2^\infty \frac{2+x}{x^3}\,dx = \int_2^\infty \frac{2}{x^3} + \frac{1}{x^2}\,dx = \left[-\frac{1}{x^2} - \frac{1}{x}\right]_2^\infty = 0 - \left[-\frac{1}{4} - \frac{1}{2}\right] = \frac{3}{4}$$

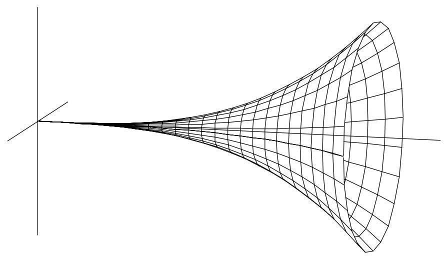
3. If it requires 12 lb of force to stretch a spring from rest to 3 ft, how much work will it take to stretch it from 3 ft to 6 ft?

- **a.** 12 ft-lb
- **b.** 18 ft-lb
- **c.** 36 ft-lb
- **d.** 54 ft-lb correctchoice
- **e.** 72 ft-lb

$F = kx \qquad 12 = k3 \qquad k = 4 \qquad F = 4x$

$$W = \int_3^6 4x\,dx = \left[2x^2\right]_3^6 = 72 - 18 = 54$$

4. The curve $y = \frac{1}{3}x^3$ for $0 \le x \le 1$ is rotated about the x-axis. Find the integral which gives the area of the surface swept out.



a. $\int_0^1 2\pi \frac{1}{3}x^3 \sqrt{1+x^4}\, dx$ correctchoice

b. $\int_0^1 \sqrt{1+x^4}\, dx$

c. $\int_0^1 2\pi x \sqrt{1+x^4}\, dx$

d. $\int_0^1 \frac{1}{3}x^3 \sqrt{1+x^4}\, dx$

e. $\int_0^1 x\sqrt{1+x^4}\, dx$

$$r = y = \frac{1}{3}x^3 \qquad \frac{dy}{dx} = x^2 \qquad ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+x^4}$$

$$A = \int_0^1 2\pi r\, ds = \int_0^1 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx = \int_0^1 2\pi \frac{1}{3}x^3\sqrt{1+x^4}\, dx$$
$$= \frac{2}{9}\sqrt{2}\pi - \frac{1}{9}\pi$$

5. Given the partial fraction expansion $\frac{x+3}{x(x+1)^2} = \frac{3}{x} - \frac{3}{x+1} - \frac{2}{(x+1)^2}$, compute $\int_1^2 \frac{x+3}{x(x+1)^2}\, dx$.

a. $3\ln 2 + 6\ln 3 + \frac{1}{3}$

b. $3\ln 2 - 3\ln 3 + \frac{2}{3}$

c. $6\ln 2 - 3\ln 3 - \frac{1}{3}$ correctchoice

d. $10\ln 2 - 7\ln 3$

e. $10\ln 2 - 7\ln 3 - \frac{1}{2}$

$$\int_1^2 \frac{x+3}{x(x+1)^2}\, dx = \int_1^2 \frac{3}{x} - \frac{3}{x+1} - \frac{2}{(x+1)^2}\, dx = \left[3\ln x - 3\ln(x+1) + \frac{2}{x+1}\right]_1^2$$
$$= \left[3\ln 2 - 3\ln 3 + \frac{2}{3}\right] - [3\ln 1 - 3\ln 2 + 1] = 6\ln 2 - 3\ln 3 - \frac{1}{3}$$

6. Use the Trapezoid Rule with $n = 3$ to approximate $\int_0^9 \frac{x}{x+3}\,dx$.

a. $\frac{9}{10}$

b. $9 - \ln 2 - \ln 5$

c. $\frac{37}{24}$

d. $\frac{33}{16}$

e. $\frac{37}{8}$ correctchoice

$\Delta x = \frac{9-0}{3} = 3 \qquad x_i = 0, 3, 6, 9 \qquad f(x) = \frac{x}{x+3}$

$T_3 = \left[\frac{1}{2}f(0) + f(3) + f(6) + \frac{1}{2}f(9)\right]\Delta x = \left[\frac{1}{2}\bullet 0 + \frac{3}{6} + \frac{6}{9} + \frac{1}{2}\bullet\frac{9}{12}\right]3 = \frac{37}{8} = 4.625$

7. The function $x(t) = Ae^{5t} + \frac{3}{5}$ satisfies which equation?

a. $\frac{dx}{dt} = 3x - 5$

b. $\frac{dx}{dt} = 5x + 3$

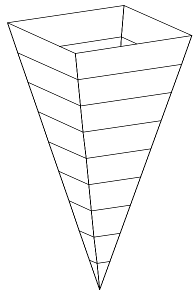
c. $\frac{dx}{dt} = 5x - 3$ correctchoice

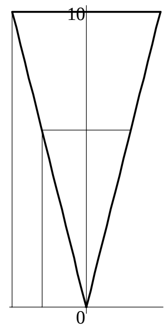
d. $\frac{dx}{dt} = -5x - 3$

e. $\frac{dx}{dt} = -3x - 5$

$x = Ae^{5t} + \frac{3}{5} \qquad \frac{dx}{dt} = 5Ae^{5t} = 5\left(x - \frac{3}{5}\right) = 5x - 3$

8. (15 points) A square pyramid with the point at the bottom is 10 cm high and the square at the top is 5 cm on a side. If the pyramid is filled with water, how much work does it take to pump the water out the top? $\rho = 1\frac{\text{gm}}{\text{cm}^3}$ $g = 980\frac{\text{cm}}{\text{sec}^2}$





$$\frac{s}{y} = \frac{5}{10} \qquad s = \frac{1}{2}y \qquad A = s^2 = \frac{1}{4}y^2$$

$$dV = \frac{1}{4}y^2 dy \qquad dF = \rho g dV = \frac{1}{4}\rho g y^2 dy \qquad D = (10-y)$$

$$W = \int_0^{10} D\, dF = \int_0^{10}(10-y)\frac{1}{4}\rho g y^2\, dy = \frac{1}{4}\rho g \int_0^{10}(10y^2 - y^3)\, dy$$

$$= \frac{1}{4}\rho g\left[\frac{10y^3}{3} - \frac{y^4}{4}\right]_0^{10} = \frac{10000}{4}\rho g\frac{1}{12} = \frac{625}{3}\rho g \text{ erg}$$

$$= \tfrac{625}{3}980 \text{ erg} = 204166.\bar{6} \text{ erg}$$

9. (15 points) Find the partial fraction expansion for $\frac{3x^2 - 10x + 4}{x(x-2)^2}$.

$$\frac{3x^2 - 10x + 4}{x(x-2)^2} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$3x^2 - 10x + 4 = A(x-2)^2 + Bx(x-2) + Cx$$

$x = 0:\quad 4 = A(4) \quad\Rightarrow\quad A = 1$

$x = 2:\quad -4 = C(2) \quad\Rightarrow\quad C = -2$

$x = 1:\quad -3 = A(1) + B(-1) + C(1) = -1 - B \quad\Rightarrow\quad B = 2$

$$\frac{3x^2 - 10x + 4}{x(x-2)^2} = \frac{1}{x} + \frac{2}{x-2} - \frac{2}{(x-2)^2}$$

10. (15 points) Solve the initial value problem $\frac{dy}{dx} = \frac{\sin x}{y^2}$ with $y(0) = 3$.

$$\int y^2 dy = \int \sin x\, dx \qquad \frac{y^3}{3} = -\cos x + C$$

$x = 0$ when $y = 3$ $\qquad 9 = -1 + C \qquad C = 10$

$$\frac{y^3}{3} = -\cos x + 10 \qquad y^3 = 30 - 3\cos x \qquad y = (30 - 3\cos x)^{1/3}$$