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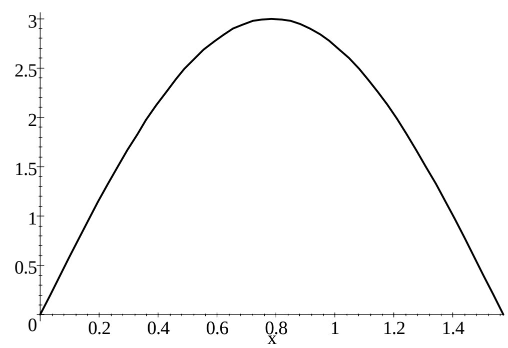
MATH 172 FINAL EXAM Fall 1999

Section 502 P. Yasskin

Multiple Choice: (5 points each)

**1.** Find the area below $y = 3\sin(2x)$ above the $x$-axis for $0 \le x \le \frac{\pi}{2}$.




**a.** $\frac{1}{2}$
**b.** $\frac{3}{2}$
**c.** $\frac{\pi}{2}$
**d.** $\frac{3\pi}{2}$
**e.** $3$

**2.** The region below $y = 3\sin(2x)$ above the $x$-axis for $0 \le x \le \frac{\pi}{2}$ is rotated about the $y$-axis. (See the figure in problem 1.) Which formula will give the volume of the solid of revolution?

**a.** $A = \int_0^{\pi/2} x^2 \sin(2x)\,dx$
**b.** $A = \int_0^{\pi/2} 3x\sin(2x)\,dx$
**c.** $A = \int_0^{\pi/2} 6\pi x\sin(2x)\,dx$
**d.** $A = \int_0^{\pi/2} 9\pi \sin^2(2x)\,dx$
**e.** $A = \int_0^{\pi/2} 18\pi \sin^2(2x)\,dx$

**3.** A $1$ m bar has linear mass density $\rho = \frac{1}{1+x^2}\ \frac{\text{kg}}{\text{m}}$ where $x$ is measured from one end. Find the total mass.

- **a.** $M = \frac{\pi}{4}$ kg
- **b.** $M = \frac{\pi}{2}$ kg
- **c.** $M = \frac{1}{2}$ kg
- **d.** $M = 45$ kg
- **e.** $M = 90$ kg

**4.** A $1$ m bar has linear mass density $\rho = \frac{1}{1+x^2}\ \frac{\text{kg}}{\text{m}}$ where $x$ is measured from one end. Find the center of mass.

- **a.** $\bar{x} = \frac{\ln 2}{90}$ m
- **b.** $\bar{x} = \frac{\ln 2}{2}$ m
- **c.** $\bar{x} = \frac{2\ln 2}{\pi}$ m
- **d.** $\bar{x} = \frac{\ln 2}{2\pi}$ m
- **e.** $\bar{x} = \frac{1}{2}$ m

**5.** Compute $\int_{-\pi/2}^{\pi/2} \sin^6\theta \cos\theta \, d\theta$

- **a.** $-\frac{2}{7}$
- **b.** $-\frac{1}{7}$
- **c.** $0$
- **d.** $\frac{1}{7}$
- **e.** $\frac{2}{7}$

**6.** The curve $y = x^3$ for $0 \le x \le 3$ is rotated about the $x$-axis. Which formula will give the area of the surface of revolution?

- **a.** $A = \int_0^3 2\pi x\sqrt{1+9x^4}\, dx$
- **b.** $A = \int_0^3 2\pi x^3\sqrt{1+9x^4}\, dx$
- **c.** $A = \int_0^3 2\pi x^3\, dx$
- **d.** $A = \int_0^3 2\pi x(3x^2)\, dx$
- **e.** $A = \int_0^3 \pi x\sqrt{1+9x^4}\, dx$

7. Compute $\int_0^2 \frac{2x}{4-x^2}\,dx$

**a.** $-\infty$
**b.** $-\ln 4$
**c.** $\frac{\pi}{4}$
**d.** $\ln 4$
**e.** $\infty$

8. If it requires $24$ J of **work** to stretch a spring from rest to $4$ m, how much work will it take to stretch it from $2$ m to $6$ m?

**a.** $6$ J
**b.** $12$ J
**c.** $24$ J
**d.** $48$ J
**e.** $96$ J

9. Which term is incorrect in the following partial fraction expansion?

$$\frac{x^3-2x+3}{(x-2)^2(x-3)(x^2+4)} = \underbrace{\frac{A}{x-2}}_{\text{a.}} + \underbrace{\frac{Bx+C}{(x-2)^2}}_{\text{b.}} + \underbrace{\frac{D}{x-3}}_{\text{c.}} + \underbrace{\frac{Ex+F}{x^2+4}}_{\text{d.}}$$

**e.** They are all correct.

10. Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{3^n}(x-2)^n$.

**a.** $0$
**b.** $\frac{1}{3}$
**c.** $3$
**d.** $9$
**e.** $\infty$

**11**. (10 points) Compute $\int_0^{\pi/2} 3x\cos(2x)\,dx$

**12**. (10 points) Find the length of the parametric curve given by $x = t^2$, $y = \frac{2}{3}t^3$, $z = \frac{1}{4}t^4$ for $0 \leq t \leq 2$.

HINT: Factor the quantity in the square root.

**13.** (10 points) Find the volume of the solid whose base is the **semi**-circle $x^2 + y^2 = 9$ for $y \geq 0$ and whose crosssections perpendicular to the $x$-axis are **squares**.

**14.** (10 points) Solve the differential equation $\dfrac{dy}{dx} = 1 + x^2 + y^2 + y^2x^2$ with the initial condition $y(3) = 0$.

**15.** (10 points) Given the series $e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \cdots,$

**a.** (5 pts) compute the series for $e^{2x}$

**b.** (5 pts) and use it to compute $\lim_{x\to 0} \frac{e^{2x} - 1 - 2x}{x^2}.$

(2 pts only for l'Hospital's Rule.)