Name________________________________

MATH 172H   Exam 2   Fall 2019

Sections 202   P. Yasskin

1-12	/60	11	/22
10	/22	Total	/104

12 Multiple Choice: (5 points each. No part credit.)

1. Find the arclength of the parametric curve $(x,y) = (2t^2, t^3)$ between $t = 0$ and $t = 1$.

a. $\dfrac{1}{27}$

b. $\dfrac{37}{27}$

c. $\dfrac{61}{27}$

d. $\dfrac{1}{27}(13^{3/2} - 8)$

e. $\dfrac{1}{27}(13^{1/2} - 4)$

2. If the parametric curve $(x,y) = (2t^2, t^3)$ between $t = 0$ and $t = 1$ is rotated about the y-axis, set up the integral for the area of the surface swept out.

a. $\int_0^1 4\pi t^2 \sqrt{4 + 9t^2}\, dt$

b. $\int_0^1 2\pi t^4 \sqrt{16 + 9t^2}\, dt$

c. $\int_0^1 4\pi t^4 \sqrt{16 + 9t^2}\, dt$

d. $\int_0^1 2\pi t^3 \sqrt{4 + 9t^2}\, dt$

e. $\int_0^1 4\pi t^3 \sqrt{16 + 9t^2}\, dt$

3. Consider the partial fraction expansion

$$\frac{2}{(x-2)(x-3)^2(x-4)} = \frac{A}{x-2} + \frac{B}{x-3} + \frac{C}{(x-3)^2} + \frac{D}{(x-4)}$$

What is $A + B + C + D$?

a. -2
b. 2
c. -3
d. 3
e. -4

4. Given the general partial fraction expansion $\frac{4(x+1)}{x^2(x+2)^2} = \frac{1}{x^2} - \frac{1}{(x+2)^2}$,

find $\int_2^4 \frac{4(x+1)}{x^2(x+2)^2}\,dx$.

a. $\frac{1}{3}$
b. $\frac{1}{4}$
c. $\frac{1}{6}$
d. $\frac{1}{12}$
e. $\frac{11}{72}$

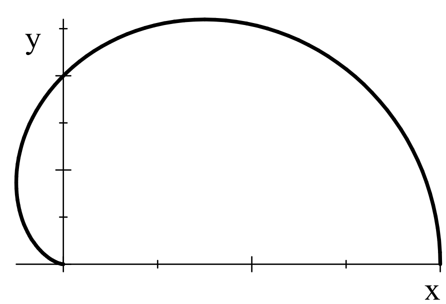
5. Compute $\int_0^2 \frac{1}{x^2+4}\,dx$.

- **a**. $\frac{\pi}{16}$
- **b**. $\frac{\pi}{8}$
- **c**. $\frac{\pi}{4}$
- **d**. π
- **e**. 2π

6. The integral $\int_1^\infty \frac{\sin^2 x}{x^3}\,dx$

- **a**. converges to $\frac{1}{2}$
- **b**. converges to 3
- **c**. converges to $\frac{3}{4}$
- **d**. converges but not to $\frac{1}{2}$, 3 nor $\frac{3}{4}$
- **e**. diverges

7. Find the area inside the upper half of the cardioid, $r = 1 + \cos\theta$.



- **a**. $\frac{\pi}{4}$
- **b**. $\frac{3\pi}{8}$
- **c**. $\frac{\pi}{2}$
- **d**. $\frac{3\pi}{4}$
- **e**. $\frac{3\pi}{2}$

8. Find the mass of a 9 cm bar whose mass density $\delta(x) = \dfrac{1}{(x+1)^2}$ if x is measured from one end of the bar.

a. $\dfrac{1}{10}$

b. $\dfrac{9}{10}$

c. $\dfrac{11}{10}$

d. $\dfrac{1}{100}$

e. $\dfrac{99}{100}$

9. Consider the series $S = \sum_{n=2}^{\infty} a_n$. If the k^{th} partial sum is $S_k = \sum_{n=2}^{k} a_n = \dfrac{n}{2n+1}$, then the sum is

a. $S = \dfrac{1}{4}$

b. $S = \dfrac{2}{5}$

c. $S = \dfrac{1}{2}$

d. $S = \dfrac{3}{5}$

e. $S = \dfrac{3}{4}$

10. Consider the series $S = \sum_{n=2}^{\infty} a_n$. If the k^{th} partial sum is $S_k = \sum_{n=2}^{k} a_n = \dfrac{n}{2n+1}$, then the a_4 term of the series is

a. $a_4 = \dfrac{1}{63}$

b. $a_4 = \dfrac{1}{99}$

c. $a_4 = \dfrac{3}{7}$

d. $a_4 = \dfrac{4}{9}$

e. $a_4 = \dfrac{5}{11}$

11. A ball is dropped from 64 inches. Each time it bounces, the it reaches half the height of the previous bounce. What is the total vertical distance it travels in an infinite number of bounces?

a. 256

b. 192

c. 128

d. 96

e. 64

12. Compute: $\displaystyle\sum_{n=1}^{\infty}\left(\sec\frac{\pi}{4n} - \sec\frac{\pi}{4(n+1)}\right)$

a. $\dfrac{1}{\sqrt{2}}$

b. $\dfrac{1}{\sqrt{2}} - 1$

c. $\sqrt{2}$

d. $\sqrt{2} - \dfrac{1}{\sqrt{2}}$

e. $\sqrt{2} - 1$

Work Out: (Points indicated. Part credit possible. Show all work.)

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13. $(22$ points$)$ Compute each limit:

a. $(6$ pts$)$ $\lim_{n\to\infty}\left(\dfrac{n^2}{n-1}-\dfrac{n^2}{n+1}\right)$

b. $(8$ pts$)$ $\lim_{n\to\infty}\left(\sqrt{n+\sqrt{n}}-\sqrt{n}\right)$

c. $(8$ pts$)$ $\lim_{n\to\infty} n^{1/n}$

14. (22 points) Consider the recursively defined sequence for which $a_1 = 4$ and $a_{n+1} = \sqrt{10a_n - 16}$.
Determine whether the limit of the sequence exists and if it exists, find the limit.
Follow these steps:

a. (3 pts) Find the first 3 terms:

$a_1 =$ ____________ $a_2 =$ ____________ $a_3 =$ ____________

b. (4 pts) Assuming the limit exists, find the possible values of the limit.

c. (5 pts) State whether you want to show the sequence is increasing or decreasing.
Prove it using induction.

d. (5 pts) State whether you want to show the sequence is bounded above or below.
Prove it using induction.

e. (5 pts) Name the theorem which implies the sequence converges or diverges. State the part of the theorem you need to apply. Apply it. State the limit you get and why.