Name_____

MATH 172H          Exam 3          Spring 2020

Sections 200                        P. Yasskin

   Multiple Choice: (Points indicated. No part credit.)

**1.** $\left(1 \text{ points}\right)$   An Aggie does not lie, cheat or steal or tolerate those who do.

True ☐                    False ☐

**2.** $\left(1 \text{ points}\right)$   Each answer is one of the following or a sum of these:

   a rational number in lowest terms, e.g. $-\dfrac{217}{5}$ which is entered as "-217/5"

   a rational number in lowest terms times $\pi$, e.g. $\dfrac{217}{5}\pi$ which is entered as "217/5pi"

   exponentials such as $e^4$ or $3^{12/5}$ which are entered as "e^4" or "3^(12/5)"
   positive infinity, $\infty$, which is entered as "infinity"
   negative infinity, $-\infty$, which is entered as "-infinity"
   convergent, which is entered as "convergent"
   divergent, which is entered as "divergent"
Do not leave any spaces. Do not use decimals.
I read this.

True ☐                    False ☐

**3.** $\left(4 \text{ points}\right)$   Compute $\displaystyle\sum_{n=1}^{\infty} \dfrac{(-1)^n 4}{2^n}$.

   **a.** $0$

   **b.** $-\dfrac{4}{3}$

   **c.** $4$

   **d.** $-4$

   **e.** $\infty$

| 1-9 | /30 | 13 | /18 |
|------|-----|-------|------|
| 10-11 | /28 | 14 | /8 |
| 12 | /18 | Total | /102 |

1

**4.** (4 points) Compute $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n}{3n-1}-\frac{n+1}{3n+2}\right)$.

   **a.** 0

   **b.** $\frac{1}{3}$

   **c.** $\frac{2}{3}$

   **d.** $\frac{1}{6}$

   **e.** $\infty$

**5.** (4 points) Compute $\displaystyle\lim_{n\to\infty}\left(\sqrt{n^6+5n^2}-\sqrt{n^6-4n^3}\right)$.

   **a.** $-\infty$

   **b.** $-4$

   **c.** 2

   **d.** 9

   **e.** $\infty$

**6.** (4 points) Compute $\displaystyle\lim_{n\to\infty}\left(\frac{1}{n^2}\right)^{\frac{4}{\ln n}}$. If divergent, enter "infinity" or "-infinity".

   **a.** $e^{-8}$

   **b.** $e^{-2}$

   **c.** $e^2$

   **d.** $e^8$

   **e.** $\infty$

**7.** $\left(4 \text{ points}\right)$  Compute $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} 2^{2n}$.  If divergent, enter "infinity" or "-infinity".

   **a.** $\sin 2$

   **b.** $\sin 2 - 1$

   **c.** $\cos 2$

   **d.** $\cos 2 - 1$

   **e.** $\infty$

**8.** $\left(4 \text{ points}\right)$  If $S = \displaystyle\sum_{n=1}^{\infty} a_n$  and  $S_k = \dfrac{k}{2k+1}$,  then

   **a.** $a_4 = \dfrac{4}{9}$

   **b.** $a_4 = \dfrac{3}{7}$

   **c.** $a_4 = \dfrac{1}{2}$

   **d.** $a_4 = \dfrac{1}{63}$

   **e.** $a_4 = \dfrac{4}{21}$

**9.** $\left(4 \text{ points}\right)$  If the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$  is approximated by the  $999^{\text{th}}$  partial sum

$S_{999} = \displaystyle\sum_{n=1}^{999} \frac{(-1)^{n+1}}{n^2} \approx 0.90154267787044571280$,  how many digits of accuracy are guaranteed in this
approximation? For example, if the error is  $|E_{999}| < 10^{-5}$,  then only the digits  $0.9015$  are accurate,
and you would answer  4.

   **a.** 4,

   **b.** 5

   **c.** 10

   **d.** 1000

   **e.** 1000000

**10.** (14 points) The series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n-1}$ can be shown to diverge by which of the following Convergence Tests? Check Yes for all that work; check No for all that don't work.

**a.** $n^{\text{th}}$-Term test for Divergence:

☐ Yes ☐ No

**b.** Integral Test:

☐ Yes ☐ No

**c.** $p$-Series Test:

☐ Yes ☐ No

**d.** Simple Comparison Test comparing to $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n}$ :

☐ Yes ☐ No

**e.** Limit Comparison Test comparing to $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n}$ :

☐ Yes ☐ No

**f.** Ratio Test:

☐ Yes ☐ No

**g.** Alternating Series Test:

☐ Yes ☐ No

4

**11.** $(14 \text{ points})$ The series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$ can be shown to converge by which of the following Convergence Tests? Check Yes for all that work; check No for all that don't work.

**a.** $n^{\text{th}}$-Term test for Divergence:

☐ Yes    ☐ No

**b.** Integral Test:

☐ Yes    ☐ No

**c.** $p$-Series Test:

☐ Yes    ☐ No

**d.** Simple Comparison Test comparing to $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2}$ :

☐ Yes    ☐ No

**e.** Limit Comparison Test comparing to $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2}$ :

☐ Yes    ☐ No

**f.** Ratio Test:

☐ Yes    ☐ No

**g.** Alternating Series Test:

☐ Yes    ☐ No

Work Out: (Points indicated. Part credit possible. Show all work.)

**12.** $\left(18 \text{ points}\right)$ Find the interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{1 + \sqrt{n}}(x - 3)^n$.

a. Find the radius of convergence and state the open interval of absolute convergence.

$$R = \underline{\quad}. \text{ Absolutely convergent on } (\underline{\quad}, \underline{\quad}).$$

b. Check the **Left** Endpoint:

$x =$ _____          The series becomes _____          Circle one:

Reasons:                                                                    Convergent

                                                                            Divergent

c. Check the **Right** Endpoint:

$x =$ _____          The series becomes _____          Circle one:

Reasons:                                                                    Convergent

                                                                            Divergent

d. State the Interval of Convergence.                    Interval= _____

6

**13.** $(18\ \text{points})$  Determine whether the recursively defined sequence $a_1 = \sqrt{6}$ and $a_{n+1} = \dfrac{(a_n)^2 + 24}{10}$ is convergent or divergent. If convergent, find the limit. If divergent, say infinity or -infinity.

a. Find the first $3$ terms:    $a_1 = $ _____    $a_2 = $ _____    $a_3 = $ _____

b. Assuming the limit $\lim\limits_{n \to \infty} a_n$ exists, find the possible limits.

c. Prove the sequence is increasing or decreasing (as appropriate).

d. Prove the sequence is bounded or unbounded above or below (as appropriate).

e. State whether the sequence is convergent or divergent and name the theorem. If convergent, determine the limit. If divergent, determine if it is infinity or -infinity.

**14.** $\left(8\text{ points}\right)$  A ball is dropped from a height of  72 feet.  Each time it bounces it reaches a height which is  $\frac{1}{3}$  of the height on the previous bounce. What is the total distance travelled by the ball (with an infinite number of bounces)?