

Name \_\_\_\_\_

MATH 172H

Final Exam

Spring 2020

Sections 200

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Multiple Choice: (Points indicated. No part credit.)

1. (1 points) An Aggie does not lie, cheat or steal or tolerate those who do.

True

False

2. (1 points) Each answer is one of the following or a sum of these:

a rational number in lowest terms, e.g.  $-\frac{217}{5}$  which is entered as "-217/5"

a rational number in lowest terms times  $\pi$ , e.g.  $\frac{217}{5}\pi$  which is entered as "217/5pi"

square roots are entered using sqrt, e.g.  $\frac{1}{2}\sqrt{3}\pi$  which is entered as "1/2sqrt(3)pi"

positive infinity,  $\infty$ , which is entered as "infinity"

negative infinity,  $-\infty$ , which is entered as "-infinity"

convergent, which is entered as "convergent"

divergent, which is entered as "divergent"

Do not leave any spaces. Do not use decimals.

I read this.

True

False

3. (5 points) Find the average value of  $f(x) = \sin x$  on the interval  $0 \leq x \leq \pi$ .

a. 2

b.  $\pi$

c.  $\frac{1}{\pi}$

d.  $\frac{2}{\pi}$

e.  $\frac{1}{2\pi}$

1-9	/37	12	/15
10	/25	13	/10
11	/15	Total	/102

4. (5 points) The base of a solid is a circle of radius 3. The cross sections perpendicular to the  $x$ -axis are squares. Find the volume of the solid.

- a.  $18\pi$
- b.  $36\pi$
- c. 36
- d. 72
- e. 144

5. (5 points) The series  $\sum_{n=2}^{\infty} \frac{(-1)^2}{n^{1/2} + n^{1/3}}$  is

- a. absolutely convergent
- b. conditionally convergent
- c. divergent
- d. absolutely divergent
- e. conditionally divergent

6. (5 points) Compute  $\lim_{x \rightarrow 0} \frac{\sin(2x^3) - 2x^3}{x^9}$ . If divergent, enter "infinity" or "-infinity".

- a.  $-\infty$
- b.  $-\frac{4}{3}$
- c.  $\frac{4}{3}$
- d.  $\frac{8}{3}$
- e.  $\infty$

7. (5 points) Compute  $\left(\frac{1}{2} - \frac{2}{3}\right) + \left(\frac{2}{3} - \frac{3}{4}\right) + \dots + \left(\frac{n}{n+1} - \frac{n+1}{n+2}\right) + \dots$

If divergent, enter "infinity" or "-infinity".

- a.  $-\frac{1}{2}$
- b.  $\frac{1}{2}$
- c. 1
- d. 2
- e.  $\infty$

8. (5 points) Compute  $\int_0^{\pi/4} \tan^4 \theta \sec^2 \theta d\theta$

- a. 125
- b. 625
- c.  $\frac{4}{5}$
- d.  $\frac{1}{25}$
- e.  $\frac{1}{5}$

9. (5 points) Estimate  $\int_0^{0.1} 3 \sin(x^2) dx$  to within an error of  $|E| < 0.0001$

HINT: Use a Maclaurin series. This answer is a decimal number.

- a.  $0.3 - \frac{(0.1)^3}{2} = 0.2995$
- b. 0.3
- c.  $3(0.1)^2 - \frac{(0.1)^6}{2} = 0.0299995$
- d.  $(0.1)^3 = .001$
- e.  $(0.1)^2 = .01$

Work Out Problems: (Show all work on paper and upload in Part 2. Enter answers here.)

10. (25 points) A paper soda cup is 8 cm tall, has a circular base of radius 4 cm and a circular top of radius 5 cm. So the sides are given by rotating the line  $x = 4 + \frac{y}{8}$  about the  $y$ -axis.

a. Find the volume of the cup.

$$V =$$

b. Find the surface area of the sides of the cup.

$$A =$$

- c. There is soda in the cup up to 7 cm. The density of the soda is  $\delta$  and the acceleration of gravity is  $g$ . A 10 cm straw is put into the cup, so that the bottom of the straw is at the bottom of the cup. Set up the integral for the work done to suck the soda through the top of the straw. Be sure to explain how you got it on your paper. Do not compute the integral.

The integral will have the form:

$$W = \boxed{c} \delta g \int_{\boxed{d}}^{\boxed{e}} (\boxed{f} - y) (\boxed{g} + \boxed{h}y)^{\boxed{i}} dy$$

Identify each of the quantities in boxes:

$$c = \underline{\quad} \quad d = \underline{\quad} \quad e = \underline{\quad} \quad f = \underline{\quad} \quad g = \underline{\quad} \quad h = \underline{\quad} \quad i = \underline{\quad}$$

11. (15 points) Find the coefficients in the partial fraction expansion

$$\frac{6x+8}{(x-2)^2(x^2+1)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{Cx+D}{x^2+1}$$

Enter your coefficients here.

$$A = \underline{\quad} \quad B = \underline{\quad} \quad C = \underline{\quad} \quad D = \underline{\quad}$$

12. (15 points) Find the 4<sup>th</sup> degree Maclaurin polynomial (Taylor polynomial at 0) for  $f(x) = \sec x$ . Its form is

$$\sec x = c_0 + c_1x + c_2x^2 + c_3x^3 + c_4x^4$$

Show your derivation on paper. (Yes, you need 4 derivatives.)

Enter your coefficients here.

$$c_0 = \underline{\quad} \quad c_1 = \underline{\quad} \quad c_2 = \underline{\quad} \quad c_3 = \underline{\quad} \quad c_4 = \underline{\quad}$$

13. (10 points) Compute  $\int_3^{3\sqrt{2}} \frac{\sqrt{x^2 - 9}}{3x} dx$ .