Name____

MATH 172H Final Spring 2021

Sections 200 P. Yasskin

1-10	/50	13	/15
11	/15	14	/15
12	/10	Total	/105

Anything above 100 is extra credit.

Multiple Choice and Short Answer: (5 Points Each)

1. Compute $\int_0^1 2x \arctan x \, dx$. HINT: $\dfrac{a}{1+a} = \dfrac{1+a}{1+a} - \dfrac{1}{1+a}$

a. $\dfrac{\pi}{4} + 1$ d. $\dfrac{\pi}{4}$ g. $\dfrac{\pi}{4} - 1$

b. $\dfrac{\pi}{2} + 1$ e. $\dfrac{\pi}{2}$ h. $\dfrac{\pi}{2} - 1$

c. $\pi + 1$ f. π i. $\pi - 1$

2. Compute $\int_0^{\pi/4} \tan x \sec^4 x \, dx$.

a. $-\dfrac{1}{4}$

b. 0

c. $\dfrac{1}{4}$

d. $\dfrac{3}{4}$

e. $\dfrac{5}{4}$

3. Compute $\int \dfrac{1}{x^2\sqrt{x^2-4}} \, dx$.

a. $\dfrac{1}{2} \operatorname{arcsec} \dfrac{x}{2} + C$

b. $\dfrac{1}{4} \operatorname{arcsec} \dfrac{x}{4} + C$

c. $\dfrac{x}{2} \operatorname{arcsec} \dfrac{x}{2} + C$

d. $\dfrac{x}{4\sqrt{x^2-4}} + C$

e. $\dfrac{\sqrt{x^2-4}}{4x} + C$

4. Find the center of mass of a 2 m bar whose density is $\delta = \dfrac{1}{x^4}$ for $2 \le x \le 4$.

a. $\dfrac{9}{7}$

b. $\dfrac{18}{7}$

c. $\dfrac{18}{9}$

d. $\dfrac{3}{32}$

e. $\dfrac{3}{16}$

5. Find the arc length of the parametric curve $\vec{r}(t) = \left(t^2, \frac{2}{3}t^3\right)$ for $0 \le t \le \sqrt{3}$.

a. 6

b. $\dfrac{16}{3}$

c. $\dfrac{14}{3}$

d. 4

e. $\dfrac{8}{3}$

6. The region between the parabola $x = 5y - y^2$ and the y-axis is rotated about the y-axis. Find the volume swept out.

a. $V = \dfrac{5^4}{3}\pi$

b. $V = \dfrac{5^4}{6}\pi$

c. $V = \dfrac{5^5}{6}\pi$

d. $V = \dfrac{5^5}{12}\pi$

e. $V = 3 \cdot 5^6\pi$

7. Solve the initial value problem

$$t^4 \frac{dy}{dt} - t^3 y = t^2 \qquad \text{with} \quad y(1) = \frac{3}{2}$$

Then $y(2) =$

a. 30

b. 15

c. $\frac{15}{2}$

d. $\frac{15}{4}$

e. $\frac{15}{8}$

8. Find a power series about $x = 0$ for $f(x) = \dfrac{2x}{(1-x^2)^2}$.

a. $\sum_{n=0}^{\infty} 2nx^{2n-1}$

b. $\sum_{n=0}^{\infty} 2x^{2n-1}$

c. $\sum_{n=0}^{\infty} 2nx^{2n+1}$

d. $\sum_{n=0}^{\infty} 2x^{2n+1}$

e. $\sum_{n=0}^{\infty} 4n^3x^{2n-1}$

f. $\sum_{n=0}^{\infty} 4n^3x^{2n+1}$

9. Compute $\lim_{n\to\infty} n^{2/n}$.

a. 0

b. 1

c. 2

d. e

e. e^2

10. Compute $\lim_{x\to 0} \dfrac{x^2 - \dfrac{x^6}{6} - \sin(x^2)}{x^{10}}$.

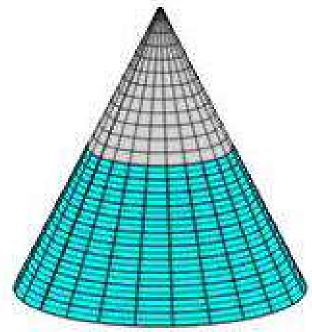
- **a**. $-\dfrac{1}{120}$
- **b**. $-\dfrac{1}{24}$
- **c**. 0
- **d**. $\dfrac{1}{24}$
- **e**. $\dfrac{1}{120}$
- **f**. ∞

Work Out: (Points indicated. Part credit possible. Show all work.)

11. (15 points) Work Out Problem

A water tank has the shape of a cone with the vertex atthe top. Its height is $H = 16$ ft and its radius is $R = 8$ ft. It is filled with salt water to a depth of 10 ft which weighs $\delta = 64 \dfrac{\text{lb}}{\text{ft}^3}$.

Find the work done to pump the water out the top of the tank.



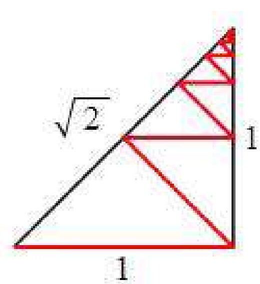
$W =$ ______________

12. (10 points) Work Out Problem

Find the length of the infinite zigzag within the $45°$ right triangle, shown at the right.

Each diagonal is at $45°$.

The total length includes the base.



$L =$ ______________

13. (15 points) Work Out Problem

A tropical fish tank contains 100 liters of salt water with initial salt concentration $35\,\frac{\text{grams}}{\text{liter}}$.

In order to reduce the concentration, salt water with concentration $25\,\frac{\text{grams}}{\text{liter}}$ is added at $2\,\frac{\text{liters}}{\text{minute}}$.

The water is kept thoroughly mixed and drained at $2\,\frac{\text{liters}}{\text{minute}}$.

How long will it take until the concentration is reduced to $30\,\frac{\text{grams}}{\text{liter}}$?

Let $S(t)$ be the number of grams of salt in the 100 liters of water at time t.

Then the concentration is $\frac{S(t)}{100}\,\frac{\text{grams}}{\text{liter}}$.

a. What is the initial condition? (Quantity not concentration.)

$S(0) =$ ______________

b. What is the differential equation?

$\frac{dS}{dt} =$ ______________

c. Solve the initial value problem.

$S(t) =$ ______________

d. Find the time when $\frac{S(t)}{100} = 30$.

$t =$ ______________

14. (15 points) Work Out Problem

Find the interval of convergence of the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt[2]{n} - \sqrt[3]{n}} \frac{(x-3)^n}{2^n}$.

a. Find the radius of convergence.

$R =$ _____________

b. Check the convergence at the left endpoint.
Be sure to name any convergence test you use and check out all conditions.

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c. Check the convergence at the right endpoint.
Be sure to name any convergence test you use and check out all conditions.

☐ Converges ☐ Diverges

d. State the interval of convergence.

Interval = _____________