

Name _____

MATH 172 Honors

Exam 1

Spring 2022

Sections 200

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Multiple Choice: (6 points each. No part credit. Circle your answers.)

1-9	/54	12	/10
10	/10	13	/10
11	/10	14	/10
		Total	/104

1. Find the area between $y = x^3 - 4x$ and $y = 5x$. (Circle your answer.)

a. 3 d. 9 g. 27 j. 81

b. $\frac{3}{2}$ e. $\frac{9}{2}$ h. $\frac{27}{2}$ k. $\frac{81}{2}$

c. $\frac{3}{4}$ f. $\frac{9}{4}$ i. $\frac{27}{4}$ l. $\frac{81}{4}$

2. Find the average value of $g(x) = x^5 - x^2$ on $[0, 3]$. (Circle your answer.)

a. 5 d. 25 g. 75 j. 225

b. $\frac{5}{2}$ e. $\frac{25}{2}$ h. $\frac{75}{2}$ k. $\frac{225}{2}$

c. $\frac{5}{4}$ f. $\frac{25}{4}$ i. $\frac{75}{4}$ l. $\frac{225}{4}$

3. Compute $\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{\arctan x}{1+x^2} dx$. (Circle your answer.)

a. $\frac{\pi}{9}$ d. $\frac{\pi}{24}$ g. $\frac{\pi^2}{9}$ j. $\frac{\pi^2}{24}$

b. $\frac{\pi}{12}$ e. $\frac{\pi}{36}$ h. $\frac{\pi^2}{12}$ k. $\frac{\pi^2}{36}$

c. $\frac{\pi}{18}$ f. $\frac{\pi}{72}$ i. $\frac{\pi^2}{18}$ l. $\frac{\pi^2}{72}$

4. Compute $\int_0^{\pi/4} \tan^2 x \sec^4 x dx$. (Circle your answer.)

- | | | | |
|-------------------|-------------------|-------------------|-------------------|
| a. $\frac{8}{15}$ | d. $\frac{4}{15}$ | g. $\frac{2}{15}$ | j. $\frac{1}{15}$ |
| b. $\frac{8}{5}$ | e. $\frac{4}{5}$ | h. $\frac{2}{5}$ | k. $\frac{1}{5}$ |
| c. $\frac{8}{3}$ | f. $\frac{4}{3}$ | i. $\frac{2}{3}$ | l. $\frac{1}{3}$ |

5. Compute $\int_0^{\pi/3} \tan \theta \sec^3 \theta d\theta$. (Circle your answer.)

- | | | | |
|------------------|------------------|---------------------|---------------------|
| a. $\frac{3}{5}$ | d. $\frac{3}{7}$ | g. $\frac{3}{5}\pi$ | j. $\frac{3}{7}\pi$ |
| b. $\frac{5}{3}$ | e. $\frac{5}{7}$ | h. $\frac{5}{3}\pi$ | k. $\frac{5}{7}\pi$ |
| c. $\frac{7}{3}$ | f. $\frac{7}{5}$ | i. $\frac{7}{3}\pi$ | l. $\frac{7}{5}\pi$ |

6. Compute $\int_0^{\pi} \sin^4 \theta \cos^4 \theta d\theta$. (Circle your answer.)

- | | | | |
|-----------------------|-----------------------|-----------------------|-----------------------|
| a. $\frac{1}{32}\pi$ | d. $\frac{3}{32}\pi$ | g. $\frac{5}{32}\pi$ | j. $\frac{7}{32}\pi$ |
| b. $\frac{1}{64}\pi$ | e. $\frac{3}{64}\pi$ | h. $\frac{5}{64}\pi$ | k. $\frac{7}{64}\pi$ |
| c. $\frac{1}{128}\pi$ | f. $\frac{3}{128}\pi$ | i. $\frac{5}{128}\pi$ | l. $\frac{7}{128}\pi$ |

7. What trig substitution should you make to do the integral $\int_0^{1/2} \frac{x^2}{\sqrt{4-9x^2}} dx$? (Circle your answer.)

a. $x = \frac{2}{3} \tan \theta$

d. $x = \frac{3}{2} \tan \theta$

b. $x = \frac{2}{3} \sin \theta$

e. $x = \frac{3}{2} \sin \theta$

c. $x = \frac{2}{3} \sec \theta$

f. $x = \frac{3}{2} \sec \theta$

8. What trig substitution should you make to do the integral $\int_4^9 \frac{x^2}{4-9x^2} dx$? (Circle your answer.)

a. $x = \frac{2}{3} \tan \theta$

d. $x = \frac{3}{2} \tan \theta$

b. $x = \frac{2}{3} \sin \theta$

e. $x = \frac{3}{2} \sin \theta$

c. $x = \frac{2}{3} \sec \theta$

f. $x = \frac{3}{2} \sec \theta$

9. Compute $\int \frac{1}{\sqrt{4x^2-1}} dx$ (Circle your answer. The $+C$ is understood.)

a. $\operatorname{arcsec}(2x) + \frac{1}{2} \ln \left| \frac{1}{2x} + \frac{1}{\sqrt{4x^2-1}} \right|$

d. $\frac{1}{2} \ln \left| \frac{1}{2x} + \frac{1}{\sqrt{4x^2-1}} \right|$

g. $-\frac{1}{2} \ln \left| \frac{1}{2x} \right|$

b. $\operatorname{arcsec}(2x) + \frac{1}{2} \ln |2x + \sqrt{4x^2-1}|$

e. $\frac{1}{2} \ln |2x + \sqrt{4x^2-1}|$

h. $\frac{1}{2} \ln \left| \frac{\sqrt{4x^2-1}}{2x} \right|$

c. $\operatorname{arcsec}(2x) + \frac{1}{2} \ln \left| \frac{2x + \sqrt{4x^2-1}}{2x} \right|$

f. $\frac{1}{2} \ln \left| \frac{2x + \sqrt{4x^2-1}}{2x} \right|$

i. $\frac{1}{4} \sqrt{4x^2-1}$

Work Out: (10 points each. Part credit possible. Show all work.)

10. Compute $\int e^{4x} \sin 3x \, dx$.

11. A bar of length π m has linear density $\delta = \sin x$ kg/m where x is measured from one end.

a. Find the total mass of the bar.

b. Find the center of mass of the bar.

12. Find the arc length of the 3D parametric curve $\vec{r}(t) = \left\langle t^3, \sqrt{\frac{3}{2}} t^2, t \right\rangle$ for $0 \leq t \leq 4$.

13. The curve $y = 2\sqrt{x}$ for $0 \leq x \leq 3$ is rotated about the x -axis. Find the surface area.

14. Consider the curve $y = f(x) = 64 - x^3$

a. Find the equation of the tangent line to $y = 64 - x^3$ at the general point $x = p$.

b. Find the area, $A(p)$, under this tangent line above the x -axis over the interval $[0, 4]$.

c. Find the value of p for which this area, $A(p)$, is a minimum.
Be sure to use the second derivative test to check it is a minimum.