Name_____

MATH 172 Honors Exam 2 Spring 2022

Sections 200 P. Yasskin

1	/8	6	/10
2	/10	7	/10
3	/10	8	/10
4	/10	9	/5
5	/10	10	/20
		Total	/103

Multiple Choice: (8 points. No part credit. Circle your answers.)

1. (8 points) Consider the general partial fraction expansion $\frac{x^3-x^2}{(x^2+4)^2} = \frac{Ax+B}{x^2+4} + \frac{Cx+D}{(x^2+4)^2}$.

Find the coefficients. (Circle 1 answer in each row.)

$A =$	-4	-3	-2	-1	0	1	2	3	4
$B =$	-4	-3	-2	-1	0	1	2	3	4
$C =$	-4	-3	-2	-1	0	1	2	3	4
$D =$	-4	-3	-2	-1	0	1	2	3	4

Work Out: (Points indicated. Part credit possible. Show all work.)

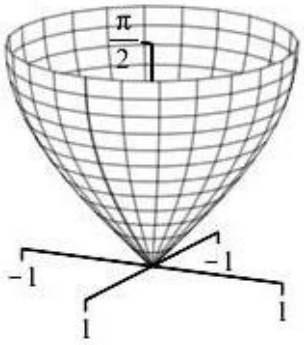
2. (10 points) Given the partial fraction expansion $\frac{2x-2}{x^4-1} = \frac{1}{x+1} + \frac{1-x}{x^2+1}$, compute $\int_0^1 \frac{2x-2}{x^4-1}\,dx$.
Simplify and evaluate all trig and inverse trig functions.

3. (10 points) Compute $\int_0^1 \frac{e^{-x}}{1-e^{-x}}\,dx$ or show why it diverges and whether it is ∞ or $-\infty$.

4. (10 points) Show why $\int_1^\infty \frac{x+\sin x}{x^{5/2}}\,dx$ converges or diverges.

5. (10 points) A cup is made by revolving the curve $x = \sin y$ about the y-axis for $0 \le y \le \frac{\pi}{2}$.

Find its volume.



6. (10 points) A cone is made by revolving the line $y = 2x$ about the y-axis for $0 \le y \le 6$ cm. It is filled with water up to a depth of 4 cm. It is sucked out a straw which reaches 3 cm above the top of the cone. How much work is done? Give your answer as a multiple of $g\delta$ where g is the acceleration of gravity and δ is the density.

7. (10 points) Solve the initial value problem:

$$\frac{dy}{dx} = \frac{x^2}{y^2} \qquad y(1) = 3$$

Find the general (explicit) solution and then find $y(0)$.

8. (10 points) Solve the initial value problem:

$$\frac{dy}{dx} = 2xy + e^{x^2} \qquad y(0) = 4$$

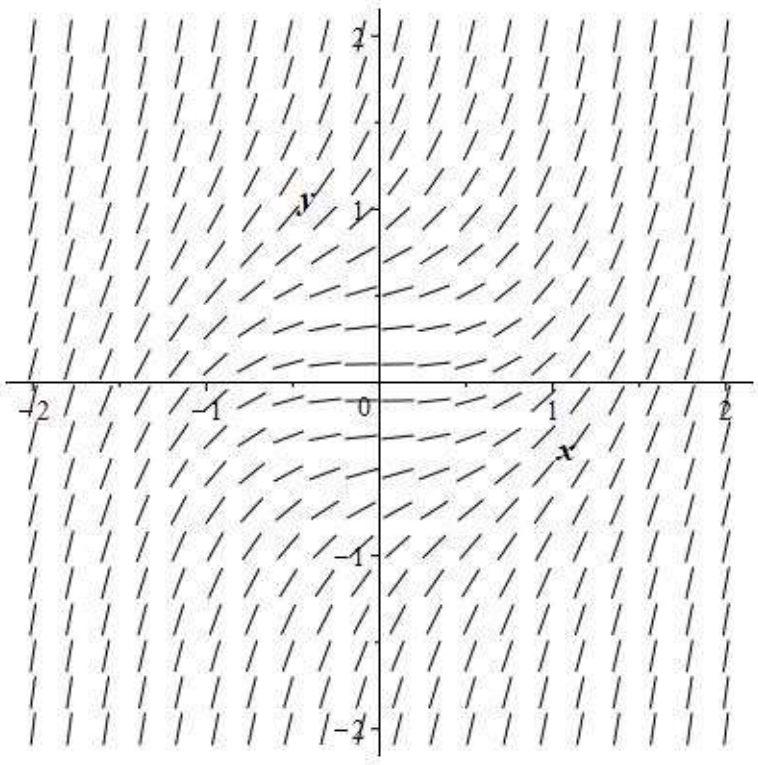
Find the general (explicit) solution and then $y(1)$.

9. (5 points) The plot at the right is the slope field for the differential equation

$$\frac{dy}{dx} = x^2 + y^2$$

On the plot, draw the solution curve satisfying the initial condition

$$y(0) = \frac{1}{2}$$



10. (20 points) A pot contains 1000 L of sugar water with a concentration of $0.01 \frac{\text{kg sugar}}{\text{L water}}$. Sugar water with a concentration of $0.04 \frac{\text{kg sugar}}{\text{L water}}$ is poured into the pot at $50 \frac{\text{L}}{\text{min}}$. The sugar water is kept mixed and drains from the tank at $50 \frac{\text{L}}{\text{min}}$.

Let $S(t)$ be the kg of sugar in the pot at time t.

a. How much sugar is in the tank at $t = 0$?

$S(0) =$ _____

b. What is the differential equation for the rate of change of $S(t)$?

$\frac{dS}{dt} =$ _____

c. How much sugar is in the pot at time t?

$S(t) =$ _____

d. Is the sugar in the pot increasing or decreasing with time?

Circle: Increasing Decreasing_