Name_____

MATH 172 Honors Exam 3 Spring 2022

Sections 200 P. Yasskin

1	/10	5	/20
2	/10	6	/10
3	/20	7	/10
4	/20	8	/10
		Total	/110

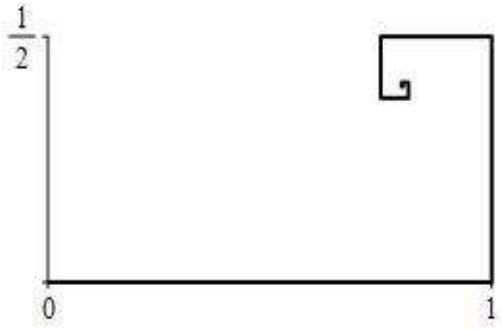
All Work Out

Points indicated. Part credit possible. Show all work.

1. (10 points) Compute $L = \lim_{n\to\infty}\left(\frac{3}{n^4}\right)^{2/\ln n}$.

2. (10 points) This rectangular spiral is made by starting at $(0,0)$, moving right by 1, up by $\frac{1}{2}$, left by $\frac{1}{4}$, down by $\frac{1}{8}$, and repeating with each step being $\frac{1}{2}$ as long the previous step.

Find the coordinates of the limit point.



3. (20 points) Determine whether each series is absolutely convergent, conditionally convergent or divergent. Be sure to name any convergence test(s) you use and check out all of its conditions:

a. $\sum_{n=0}^{\infty} \frac{n^2 + \ln n}{n^3 + \ln n}$

b. $\sum_{n=1}^{\infty} \frac{2n + 3}{(n^2 + 3n)^2}$

c. $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{n + 1}{n - 1}$

d. $\sum_{n=1}^{\infty} (-1)^n \frac{2}{n^{3/4}}$

4. (20 points) Consider the sequence recursively defined by $a_{n+1} = 5 - \frac{4}{a_n}$ starting from $a_1 = 2$. Prove the limit exist and find it. (You may assume $a_n > 0$ without proof.)

a. Write out the first 3 terms:

$a_1 =$ $a_2 =$ $a_3 =$

b. Assuming the limit exists, find the possible values.

c. What do you need to prove?

Circle one: increasing decreasing

Circle one and fill in the blank: bounded above by _____ bounded below by _____

d. Prove it is bounded above or below:

e. Prove it is increasing or decreasing:

f. What do you conclude. What Theorem did you use?

5. (20 points) Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{n}{(n^2+1)2^n}(x-6)^n$.

a. Find the radius of convergence and state the open interval of absolute convergence.

$R =$ ____. Absolutely convergent on (_____,_____).

b. Check the **Left** Endpoint:

$x =$_____ Write the series:_____ Circle one:

Reasons: Convergent

Divergent

c. Check the **Right** Endpoint:

$x =$_____ Write the series:_____ Circle one:

Reasons: Convergent

Divergent

d. State the Interval of Convergence. Interval= _____

6. (10 points) Compute $\sum_{n=1}^{\infty}\left[\sec\left(\frac{1}{n}\right)-\sec\left(\frac{1}{n+1}\right)\right]$.

7. (10 points) Find the Maclaurin series for $f(x)=\frac{\sin(x^2)}{x}$.
Give the answer in both summation form and $\cdots$ form with at least 3 terms.
Then find $f^{(9)}(0)$, the 9^{th} derivative at 0.

8. (10 points) Compute $\lim_{x\to 0}\frac{\cos(x^2)-1+\frac{x^4}{2}}{x^8}$