Name____________________________

MATH 221 Exam 2 Fall 2009

Section 503 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

| | | | |
|---|---|---|---|
| 1-13 | /65 | 15 | /10 |
| 14 | /15 | 16 | /15 |
| | | Total | /105 |

**1.** Compute $\int_0^2 \int_{-x}^{x} 3y^2\,dy\,dx$.

**a.** $8$

**b.** $12$

**c.** $16$

**d.** $24$

**e.** $0$

**2.** Compute $\int_0^9 \int_{\sqrt{y}}^{3} \pi \sin(\pi x^3)\,dx\,dy$. HINT: Reverse the order of integration.

**a.** $\dfrac{1}{9}$

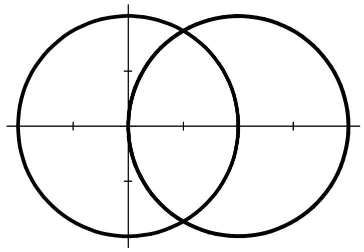
**b.** $\dfrac{2\pi}{9}$

**c.** $\dfrac{1}{3}$

**d.** $\dfrac{2}{3}$

**e.** $\dfrac{2\pi}{3}$

**3**. Find the area inside the circle $r = 2\cos\theta$ but outside the circle $r = 1$.



**a**. $\frac{\pi}{3} - \cos\frac{\pi}{3}$

**b**. $\frac{2\pi}{3} + \cos\frac{2\pi}{3}$

**c**. $\frac{\pi}{3} + \sin\frac{2\pi}{3}$

**d**. $\frac{2\pi}{3} - \sin\frac{\pi}{3}$

**e**. $\frac{2\pi}{3} + \sin\frac{\pi}{3}$

**4**. Compute $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{x^2+y^2}\, dy\, dx$. HINT: Switch to polar coordinates.

**a**. $\frac{\pi}{2}e^2$

**b**. $\frac{\pi}{2}e^4$

**c**. $\frac{\pi}{2}(e^2 - 1)$

**d**. $\frac{\pi}{2}(e^4 - 1)$

**e**. $\frac{\pi}{4}(e^4 - 1)$

**5**. Compute the mass of the solid cone $\sqrt{x^2+y^2} \le z \le 4$ if the volume density is $\rho = z$.

**a**. $4\pi$

**b**. $8\pi$

**c**. $16\pi$

**d**. $32\pi$

**e**. $64\pi$

**6**. Compute the center of mass of the solid cone $\sqrt{x^2+y^2} \le z \le 4$ if the volume density is $\rho = z$.

**a**. $\left(0, 0, \frac{5}{16}\right)$

**b**. $\left(0, 0, \frac{16}{5}\right)$

**c**. $\left(0, 0, \frac{1024\pi}{5}\right)$

**d**. $\left(0, 0, \frac{5}{1024\pi}\right)$

**e**. $\left(0, 0, \frac{8}{5}\right)$

**7**. Find the average value of the function $f = \dfrac{1}{x^2+y^2+z^2}$ over the solid region between the two spheres $x^2+y^2+z^2 = 1$ and $x^2+y^2+z^2 = 4$. HINT: $f_{\text{ave}} = \dfrac{\iiint f\,dV}{\iiint 1\,dV}$

**a**. $\frac{3}{4}$

**b**. $\frac{5}{8}$

**c**. $\frac{3}{7}$

**d**. $4\pi$

**e**. $8\pi$

**8**. Compute $\iint \cos\left(\frac{x^2}{9} + \frac{y^2}{4}\right) dx\,dy$ over the region inside the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

HINT: Elliptical coordinates are $x = 3t\cos\theta$, $y = 2t\sin\theta$.

**a**. $6\pi\sin(1) - 6\pi$

**b**. $6\pi\sin(1)$

**c**. $6\pi - 6\pi\cos(1)$

**d**. $-6\pi\cos(1)$

**e**. $6\pi\cos(1)$

**9**. Compute $\int_{(0,0)}^{(\sqrt{2},2)} x\,ds$ along the parabola $y = x^2$ parametrized as $\vec{r}(t) = (t, t^2)$.

**a**. $\frac{13}{6}$

**b**. $\frac{9}{4}$

**c**. $\frac{27}{4}$

**d**. $\frac{1}{12}(17^{3/2} - 1)$

**e**. $\frac{17^{3/2}}{12}$

**10**. Compute $\int_{(0,0)}^{(\sqrt{2},2)} \vec{\nabla} f \cdot d\vec{s}$ for $f = xy$ along the parabola $y = x^2$ parametrized as $\vec{r}(t) = (t, t^2)$.

**a**. $2$

**b**. $2\sqrt{2}$

**c**. $4$

**d**. $4\sqrt{2}$

**e**. $8$

**11**. Compute $\int \vec{F} \cdot d\vec{s}$ for $\vec{F} = (-yz^2, xz^2, z^3)$ counterclockwise around the circle $x^2 + y^2 = 4$ with $z = 4$. HINT: Parametrize the circle.

**a**. $64$

**b**. $128$

**c**. $64\pi$

**d**. $128\pi$

**e**. $256\pi$

**12**. Compute $\iiint \vec{\nabla} \cdot \vec{F}\, dV$ for $\vec{F} = (xy^2, yz^2, zx^2)$ over the solid hemisphere $x^2 + y^2 + z^2 \le 25$ with $z \ge 0$.

**a**. $\frac{125}{3}\pi$

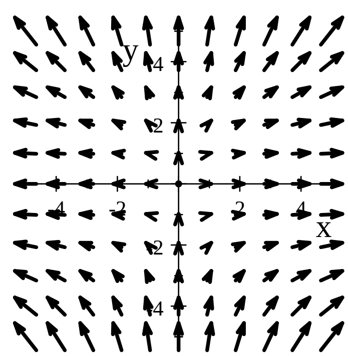
**b**. $\frac{250}{3}\pi$

**c**. $\frac{500}{3}\pi$

**d**. $1250\pi$

**e**. $2500\pi$

**13**. Which of the following vector fields are plotted at the right?




**a**. $(y^2, 4x)$

**b**. $(-y^2, 4x)$

**c**. $(4x, y^2)$

**d**. $(4x, -y^2)$

**e**. $(4xz, 4y)$

Work Out: (Points indicated. Part credit possible. Show all work.)

**14**. (15 points) Find the volume of the largest box such the length plus twice the width plus three times the height is $36$.

**15**. (10 points) Find the area of the surface $\vec{R}(p,q) = (p, p+q^2, p-q^2)$ for $0 \le p \le 3$ and $0 \le q \le 2$.

$\vec{e}_p =$

$\vec{e}_q =$

$\vec{N} =$

$\left|\vec{N}\right| =$

$A =$

**16**. (15 points) Compute $\iint_P \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ for $\vec{F} = (-yz^2, xz^2, z^3)$ over the paraboloid $z = x^2 + y^2$ with $z \le 4$, oriented **up and in**, and parametrized by $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2)$.

$\vec{\nabla} \times \vec{F} =$

$\left(\vec{\nabla} \times \vec{F}\right)\left(\vec{R}(r,\theta)\right) =$

$\vec{e}_r =$

$\vec{e}_\theta =$

$\vec{N} =$

$\iint_P \vec{\nabla} \times \vec{F} \cdot d\vec{S} =$