Name________________________ Sec_________

MATH 221 Exam 1 Spring 2011

Section 500 P. Yasskin

1-10	/50	14	/11
11	/11	15	/11
12	/11		
13	/11	Total	/105

Multiple Choice: (5 points each. No part credit.)

1. Consider the line $X = P + t\vec{v}$ where $P = (2,3,2)$ and $\vec{v} = (2,-1,2)$.
Drop a perpendicular from the point $Q = (-1,0,5)$ to a point R on the line. Then $R =$
HINT: Draw a figure.

a. $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$

b. $\left(\frac{8}{3}, \frac{8}{3}, \frac{8}{3}\right)$

c. $\left(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right)$

d. $(4,2,4)$

e. $\left(\frac{8}{3}, \frac{10}{3}, \frac{8}{3}\right)$

2. If $\vec{u}$ is 5 cm long and points $30°$ WEST of NORTH and $\vec{v}$ is 4 cm long and points $30°$ EAST of NORTH, then $\vec{u} \times \vec{v}$ is

a. 10 cm long and points DOWN.

b. 10 cm long and points UP.

c. 10 cm long and points SOUTH.

d. $10\sqrt{3}$ cm long and points DOWN.

e. $10\sqrt{3}$ cm long and points SOUTH.

3. Find the point where the line $(x,y,z) = (3-2t, 2-t, 1+t)$ intersects the plane $x + y + 3z = 2$.
At this point, $x + y + z =$

a. 2

b. 4

c. 6

d. 8

e. The line does not intersect the plane.

4. The graph of the equation $x^2 + 4x - y^2 + 4y + z^2 + 2z = -1$ is a

a. hyperboloid of one sheet
b. hyperboloid of two sheets
c. cone
d. hyperbolic paraboloid
e. hyperbolic cylinder

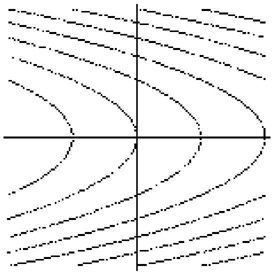
5. For the helix $\vec{r}(t) = (3t, \sin(4t), \cos(4t))$, which of the following is FALSE?

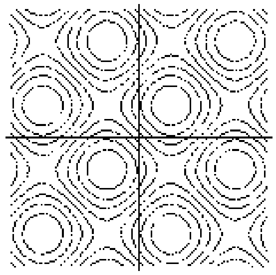
a. $\vec{v} = (3, 4\cos(4t), -4\sin(4t))$
b. $\vec{a} = (0, -16\sin(4t), -16\cos(4t))$
c. $\vec{j} = (0, -64\cos(4t), 64\sin(4t))$
d. speed $= 25$
e. arclength between $(0,0,1)$ and $(3\pi, 0, 1)$ is 5π

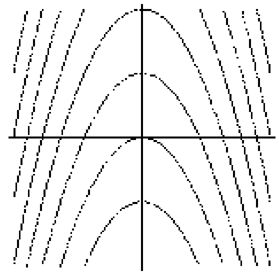
6. For the helix $\vec{r}(t) = (3t, \sin(4t), \cos(4t))$, which of the following is FALSE?

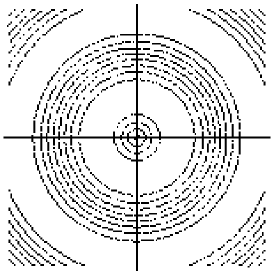
a. $\hat{T} = \left(\frac{3}{5}, \frac{4}{5}\cos(4t), -\frac{4}{5}\sin(4t)\right)$
b. $\hat{N} = (0, -\sin(4t), -\cos(4t))$
c. $\hat{B} = \left(-\frac{4}{5}, -\frac{3}{5}\cos(4t), -\frac{3}{5}\sin(4t)\right)$
d. $a_T = 0$
e. $a_N = 16$

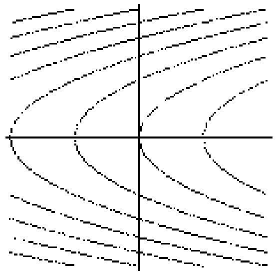
7. Which of the following is the contour plot of $f(x,y) = y^2 + x + 1$?

a. 

b. 

c. 

d. 

e. 

8. If $P(2,3) = 5$ and $\frac{\partial P}{\partial x}(2,3) = 0.4$ and $\frac{\partial P}{\partial y}(2,3) = -0.3,$ estimate $P(2.1, 2.8).$

a. 4.9
b. 4.98
c. 4.99
d. 5.01
e. 5.1

9. Currently for a certain box, the length L is 5 cm and increasing at 0.2 cm/sec, the width W is 4 cm and decreasing at 0.3 cm/sec, the height H is 3 cm and increasing at 0.1 cm/sec. Then currently, the volume V is

a. increasing at 0.1 cm/sec.
b. decreasing at 0.1 cm/sec.
c. increasing at 0.2 cm/sec.
d. decreasing at 0.2 cm/sec.
e. increasing at 0.3 cm/sec.

10. The temperature of a frying pan is $T = \dfrac{1}{1 + x^2 + 4y^2}$. An ant is located at $(2, 1)$. In what **unit vector** direction should the ant move to **decrease** the temperature as fast as possible?

a. $(-1, -2)$

b. $(1, 2)$

c. $(1, -2)$

d. $\left(\dfrac{-1}{\sqrt{5}}, \dfrac{-2}{\sqrt{5}}\right)$

e. $\left(\dfrac{1}{\sqrt{5}}, \dfrac{2}{\sqrt{5}}\right)$

Work Out: (Points indicated. Part credit possible. Show all work.)

11. (11 points) Find the mass of the helical wire $\vec{r}(t) = (3t, \sin(4t), \cos(4t))$ from $(0,0,1)$ to $(3\pi,0,1)$ if its linear density is $\rho = x^2 + y^2 + z^2$.

12. (11 points) A bead slides along the helix $\vec{r}(t) = (3t, \sin(4t), \cos(4t))$ from $(0,0,1)$ to $(3\pi,0,1)$ under the action of the force $\vec{F} = (x, xy, xz)$. Find the work done.

13. (11 points) Find the plane tangent to the graph of the function $z = x^2y + y^3x$ at the point $(x,y) = (2,1)$. Find the z-intercept.

14. (11 points) Find the plane tangent to the level surface $x\sin z + y\cos z = 3$ at the point $(x,y,z) = \left(3, 2, \frac{\pi}{2}\right)$. Find the z-intercept.

15. (11 points) Determine whether or not each of these limits exists. If it exists, find its value.

a. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+3y^6}$

b. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^6+y^6}{(x^2+y^2)^2}$

c. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x+xy^2}{x+x^3}$