Name_________________________ Sec_________

1-12	/48	14	/16
13	/16	15	/25
		Total	/105

MATH 221 Final Exam Spring 2011

Section 500 P. Yasskin

Multiple Choice: (4 points each. No part credit.)

1. Find the angle between the vectors $\vec{u} = (12,-3,4)$ and $\vec{v} = (2,1,-2)$.

a. $\frac{\pi}{6}$

b. $\frac{\pi}{4}$

c. $\frac{\pi}{3}$

d. $\frac{2\pi}{3}$

e. $\cos^{-1}\left(\frac{1}{3}\right)$

2. If $\vec{u}$ points DOWN and $\vec{v}$ points NORTHEAST then $\vec{u}\times\vec{v}$ points

a. NORTHWEST

b. SOUTHWEST

c. SOUTHEAST

d. UP

e. WEST

3. Find the point where the lines $(x,y,z) = (3-t,2+t,2t)$ and $(x,y,z) = (-1+2t,5-t,3+t)$ intersect. At this point $x+y+z =$

a. $\frac{17}{2}$

b. 9

c. $\frac{25}{3}$

d. 11

e. They do not intersect.

4. For the curve $\vec{r}(t) = (t^2, 2t, \ln t)$, compute $\hat{B} = \dfrac{\vec{v} \times \vec{a}}{|\vec{v} \times \vec{a}|}$.

a. $\left(\dfrac{-1}{2t^2+1}, \dfrac{-2t}{2t^2+1}, \dfrac{2t^2}{2t^2+1} \right)$

b. $\left(\dfrac{1}{2t^2+1}, \dfrac{-2t}{2t^2+1}, \dfrac{2t^2}{2t^2+1} \right)$

c. $\left(\dfrac{1}{2t^2+1}, \dfrac{2t}{2t^2+1}, \dfrac{2t^2}{2t^2+1} \right)$

d. $\left(\dfrac{-1}{2t^2+1}, \dfrac{-2t}{2t^2+1}, \dfrac{-2t^2}{2t^2+1} \right)$

e. $\left(\dfrac{-1}{2t^2+1}, \dfrac{2t}{2t^2+1}, \dfrac{-2t^2}{2t^2+1} \right)$

5. Find the equation of the plane tangent to the graph of $z = 3x^2y - 2y^3$ at the point $(2, 1)$. The z-intercept is

a. -20
b. -14
c. 14
d. 20
e. 40

6. Find the equation of the line perpendicular to the graph of $x^3y^2z - 2x^2z^2 = 10$ at the point $(1, 3, 2)$. This line intersects the xy-plane at

a. $\left(-\dfrac{19}{3}, 2, 0 \right)$

b. $\left(2, \dfrac{19}{3}, 0 \right)$

c. $\left(\dfrac{19}{3}, -2, 0 \right)$

d. $(-75, -21, 0)$

e. $(21, -75, 0)$

7. Han Deut is flying the Millennium Eagle through a dangerous zenithon field whose density is $\rho = xyz$. If his current position is $\vec{r} = (2, 1, -1)$, and his current velocity is $\vec{v} = (3, 2, 1)$ find the current rate of change of the density.

a. -5

b. -1

c. 1

d. 3

e. 18

8. Compute $\iint xy \, dA$ over the region between $y = x^2$ and $y = 3x$.

a. $\frac{3^3}{4}$

b. $\frac{3^4}{5}$

c. $\frac{3^5}{4}$

d. $\frac{3^5}{8}$

e. $\frac{3^6}{2}$

9. Compute $\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \cos(x^2 + y^2) \, dx \, dy$.
HINT: Switch to polar coordinates.

a. $\pi \cos(4)$

b. $\pi \cos(4) - \pi$

c. $\frac{\pi}{2} \sin(4)$

d. $\frac{\pi}{2} \sin(4) - \frac{\pi}{2}$

e. $2\pi \cos(4) - 2\pi$

10. Compute $\iiint z\,dV$ over the volume above the paraboloid $z = x^2 + y^2$ below $z = 4$.

a. $\frac{64\pi}{3}$
b. $\frac{32\pi}{3}$
c. $\frac{16\pi}{5}$
d. 21π
e. 11π

11. Compute $\int \vec{F} \cdot d\vec{s}$ for $\vec{F} = (y+z, x+z, x+y)$ along the curve $\vec{r}(t) = \left(2\frac{t^2+2}{t+1}, 3\frac{t^2+4}{t+2}, 4\frac{t^2+6}{t+3}\right)$ from $(4,6,8)$ to $(3,5,7)$.
HINT: Find a scalar potential.

a. 33
b. 6
c. 0
d. -6
e. -33

12. Compute $\oint \vec{F} \cdot d\vec{s}$ for $\vec{F} = (\sin(x^3) - 4y, \tan(y^5) + 6x)$ counterclockwise around the triangle with vertices $(0,0)$, $(2,0)$ and $(0,6)$.
Hint: Use Green's Theorem.

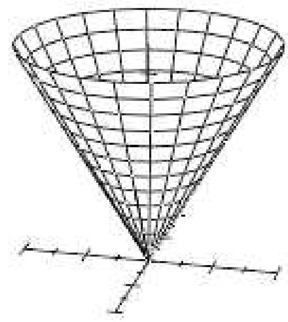
a. 6
b. 12
c. 30
d. 60
e. 80

Work Out: (Points indicated. Part credit possible. Show all work.)

13. (16 points) Use Lagrange multipliers to find 4 numbers, a, b, c, and d, for which $a + 2b + 3c + 4d = 48$ and whose product is a maximum.

14. (16 points) Find the mass and the y-component of the center of mass of the quarter of the sphere $x^2 + y^2 + z^2 \le 4$ in the first octant $(x \ge 0, y \ge 0, z \ge 0)$ if the mass density is $\delta = xyz$.

15. (25 points) Verify Stokes' Theorem $\iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint_{\partial C} \vec{F} \cdot d\vec{s}$



for the vector field $\vec{F} = (yz, -xz, z^2)$ and the cone

$z = 2\sqrt{x^2 + y^2} \le 2$ oriented down and out.

Be careful with orientations. Use the following steps:

First the Left Hand Side:

a. Compute the curl:

$\vec{\nabla} \times \vec{F} =$

b. Complete the parametrization of the surface C:

$\vec{R}(r,\theta) = \left(r\cos\theta, r\sin\theta, \underline{\hspace{3cm}} \right)$

c. Compute the tangent vectors:

$\vec{e}_r =$

$\vec{e}_\theta =$

d. Compute the normal vector:

$\vec{N} =$

e. Evaluate $\vec{\nabla} \times \vec{F}$ on the surface:

$\vec{\nabla} \times \vec{F} \Big|_{\vec{R}(r,\theta)} =$

f. Compute the dot product:

$\vec{\nabla} \times \vec{F} \cdot \vec{N} =$

g. Find the limits of integration:

h. Compute the left hand side:

$\iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S} =$

Second the Right Hand Side:

i. Parametrize the boundary circle ∂C:

$\vec{r}(\theta) =$

j. Compute the tangent vector:

$\vec{v} =$

k. Evaluate $\vec{F} = (yz, -xz, z^2)$ on the curve:

$\vec{F}\Big|_{\vec{r}(\theta)} =$

l. Compute the dot product:

$\vec{F} \cdot \vec{v} =$

m. Compute the right hand side:

$\oint_{\partial C} \vec{F} \cdot d\vec{s} =$