

Name _____

MATH 221
Sections 503

Exam 2 Fall 2012
P. Yasskin

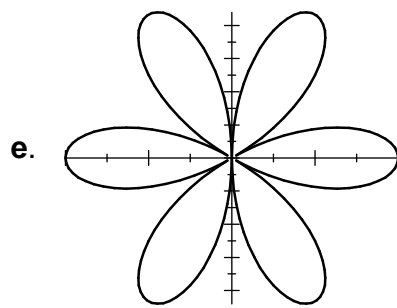
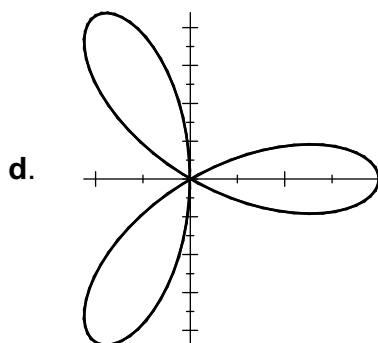
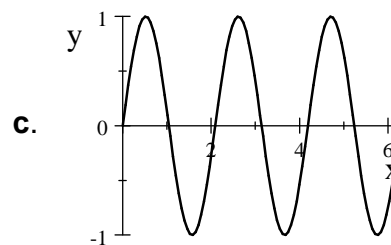
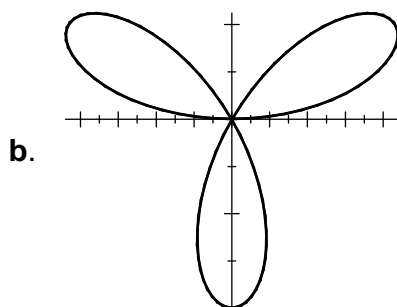
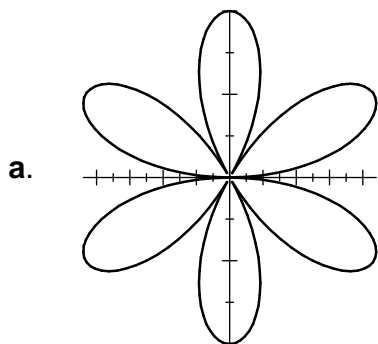
1-8	/48
9	/12
10	/20
11	/20
Total	/100

Multiple Choice: (6 points each. No part credit.)

1. Compute $\int_0^2 \int_x^2 5y^3 dy dx$.

- a. 68
- b. 48
- c. 32
- d. 27
- e. 15

2. Which of the following is the polar plot of $r = \sin(3\theta)$?



3. Find the mass of a triangular plate whose vertices are $(0,0)$, $(1,0)$ and $(1,3)$, if the density is $\rho = 2y$.

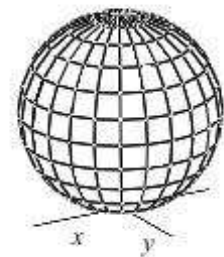
- a. 1
- b. 2
- c. 3
- d. 4
- e. 5

4. Find the y -component of the center of mass of a triangular plate whose vertices are $(0,0)$, $(1,0)$ and $(1,3)$, if the density is $\rho = 2y$.

- a. $\frac{9}{2}$
- b. $\frac{7}{2}$
- c. $\frac{5}{2}$
- d. $\frac{3}{2}$
- e. $\frac{1}{2}$

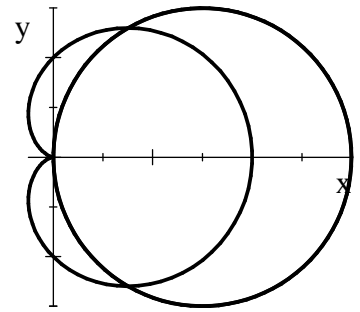
5. In spherical coordinates $\rho = 2 \cos \varphi$ is a sphere of radius 1 centered at $(0,0,1)$. If its volume density is $\delta = x^2 + y^2 + z^2$ then its mass is given by the integral:

- a. $M = \int_0^{2\pi} \int_0^{\pi/2} \int_0^{\pi} \rho^2 2 \cos \varphi \sin \varphi d\rho d\varphi d\theta$
- b. $M = \int_0^{2\pi} \int_0^{\pi} \int_0^{2 \cos \varphi} \rho^4 \sin \varphi d\rho d\varphi d\theta$
- c. $M = \int_0^{2\pi} \int_0^{\pi/2} \int_0^{2 \cos \varphi} \rho^2 \sin \varphi d\rho d\varphi d\theta$
- d. $M = \int_0^{2\pi} \int_0^{\pi} \int_0^{2 \cos \varphi} \rho^2 \sin \varphi d\rho d\varphi d\theta$
- e. $M = \int_0^{2\pi} \int_0^{\pi/2} \int_0^{2 \cos \varphi} \rho^4 \sin \varphi d\rho d\varphi d\theta$



6. Find the area inside the circle $r = 3 \cos \theta$ and outside the cardioid $r = 1 + \cos \theta$.

- a. $\frac{\pi}{4}$
- b. $\frac{\pi}{2}$
- c. π
- d. $\frac{3\pi}{2}$
- e. 2π



7. Parabolic coordinates are given by $u = y - x^2$ and $v = y + x^2$ where $v > u$. So the area element is $dA = dx dy =$

- a. $\frac{1}{2\sqrt{2} \sqrt{v-u}} du dv$
- b. $\frac{-1}{2\sqrt{2} \sqrt{v-u}} du dv$
- c. $\frac{1}{4\sqrt{2} \sqrt{v-u}} du dv$
- d. $\frac{-1}{4\sqrt{2} \sqrt{v-u}} du dv$
- e. $\frac{1}{8\sqrt{2} \sqrt{v-u}} du dv$

8. If $f = xe^{yz} - ye^{xz}$, then $\vec{\nabla} \times \vec{\nabla}f =$

- a. $(2xe^{yz} - 2xe^{xz} + 2xyze^{yz}, 0, 2ze^{xz} - 2ze^{yz})$
- b. $(2xe^{yz} + 2xe^{xz} + 2xyze^{yz}, 0, 2ze^{xz} + 2ze^{yz})$
- c. $(e^{yz} - yze^{xz}, -xze^{yz} + e^{xz}, xye^{yz} - xye^{xz})$
- d. $(e^{yz} - yze^{xz}, xze^{yz} - e^{xz}, xye^{yz} - xye^{xz})$
- e. $\vec{0}$

Work Out: (Points indicated. Part credit possible. Show all work.)

9. (12 points) Determine whether or not each of these limits exists. If it exists, find its value.

a. $\lim_{(x,y) \rightarrow (0,0)} \frac{3x^2y^2}{x^6 + 3y^3}$

b. $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + y^2}$

10. (20 points) Compute $\iint \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ for the vector field $\vec{F} = (yz, -xz, z^2)$ over the paraboloid $z = 9 - x^2 - y^2$ for $z \geq 5$ oriented down and in.

Note: The paraboloid may be parametrized as $\vec{R}(r, \theta) = (r \cos \theta, r \sin \theta, 9 - r^2)$.

11. (20 points) Compute $\iiint \vec{\nabla} \cdot \vec{F} dV$ for the vector field $\vec{F} = (x^3, y^3, x^2z + y^2z)$ over the solid region below the cone $z = 9 - \sqrt{x^2 + y^2}$ and above the plane $z = 5$.