1-9	/45
10	/15
11	/25
12	/15
Total	/100

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MATH 221 Final Exam Fall 2012

Sections 502 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. If $\vec{a} = (3,3,-1)$ and $\vec{b} = (2,5,1)$ then $\left|\vec{a} - 3\vec{b}\right| =$

a. 26

b. 13

c. 0

d. -13

e. -26

2. Find the point on the elliptic paraboloid $z - \dfrac{x^2}{2} - \dfrac{y^2}{4} = 1$ where a unit normal is $\left(\dfrac{2}{3}, \dfrac{1}{3}, -\dfrac{2}{3}\right)$.

a. $(-2,-4,7)$

b. $(2,4,7)$

c. $(4,2,10)$

d. $\left(1,1,\dfrac{7}{4}\right)$

e. $\left(-1,-1,\dfrac{7}{4}\right)$

3. Find the point on the curve $\vec{r}(t) = \left(t, t^2, \frac{2}{3}t^3\right)$ where the unit tangent is $\hat{T} = \left(\frac{1}{5}, \frac{2\sqrt{2}}{5}, \frac{4}{5}\right)$.

a. $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{12}\right)$

b. $\left(1, 1, \frac{2}{3}\right)$

c. $\left(\sqrt{2}, 2, \frac{4}{3}\sqrt{2}\right)$

d. $\left(2, 4, \frac{16}{3}\right)$

e. $\left(-1, 1, -\frac{2}{3}\right)$

4. Compute $\int_0^8 \int_{y^{1/3}}^2 \cos(x^4)\, dx\, dy$

HINT: Plot the region of integration and reverse the order of integration.

a. $\frac{1}{4}\sin(64)$

b. $\frac{1}{4}\cos(64)$

c. $\frac{1}{4}\sin(16) - \frac{1}{4}$

d. $\frac{1}{4}\cos(16) - \frac{1}{4}$

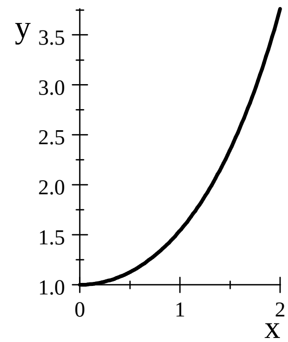
e. $\frac{1}{4}\sin(16)$

5. Find the y-component of the centroid (center of mass with density 1) of the hyperbolic cosine curve $y = \cosh x$ for $0 \le x \le 2$.
HINTS: Parametrize the curve as $\vec{r}(t) = (t, \cosh t)$.

$\cosh x = \dfrac{e^x + e^{-x}}{2}$ $\quad \sinh x = \dfrac{e^x - e^{-x}}{2}$ $\quad \cosh^2 x - \sinh^2 x = 1$

$\sinh 0 = 0$ $\quad \cosh 0 = 1$ $\quad (\sinh x)' = \cosh x$ $\quad (\cosh x)' = \sinh x$

$\sinh^2 x = \dfrac{\cosh 2x - 1}{2}$ $\quad \cosh^2 x = \dfrac{\cosh 2x + 1}{2}$



a. $\bar{y} = \dfrac{4 + \sinh 4}{4 \sinh 2}$

b. $\bar{y} = \dfrac{1 + \cosh 4}{2 \sinh 2}$

c. $\bar{y} = \dfrac{4 + \cosh 4}{2 \sinh 2}$

d. $\bar{y} = \dfrac{\cosh 4 - 1}{2 \sinh 2}$

e. $\bar{y} = \dfrac{1 + \sinh 4}{2 \sinh 2}$

6. Find the line perpendicular to the surface $y^2 z - 2xz = 6$ at the point $(1, 2, 4)$.
This line intersects the xy-plane at

a. $(-17, 30, 0)$

b. $(17, -30, 0)$

c. $(-15, 34, 0)$

d. $(15, -34, 0)$

e. The line does not intersect the xy-plane.

7. Let $L = \lim_{(x,y)\to(0,0)} \dfrac{e^{(x^2+y^2)} - 1}{x^2 + y^2}$

a. L does not exist by looking at the paths $y = x$ and $y = -x$.
b. L exists and $L = 1$ by looking at the paths $y = mx$.
c. L does not exist by looking at polar coordinates.
d. L exists and $L = 1$ by looking at polar coordinates.
e. L exists and $L = 0$ by looking at polar coordinates.

8. Compute $\oint \vec{F} \cdot d\vec{s}$ for $\vec{F} = (-x^2 y, xy^2)$ along the semicircle $y = \sqrt{16 - x^2}$ from $(4,0)$ to $(-4,0)$ followed by the line segment from $(-4,0)$ back to $(4,0)$.
HINT: Use Green's Theorem.

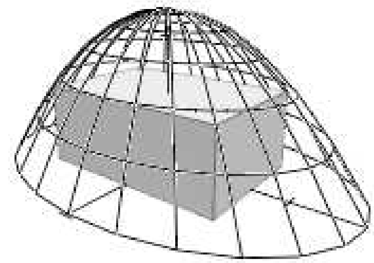
a. 0
b. $\dfrac{64}{5}\pi$
c. $\dfrac{64}{3}\pi$
d. 32π
e. 64π

9. Compute $\oint \vec{F} \cdot d\vec{s}$ for $\vec{F} = (2xy + y^2, x^2 + 2xy)$ along the parabola $y = x^2$ from $(1,1)$ to $(2,4)$.
HINT: Find a scalar potential.

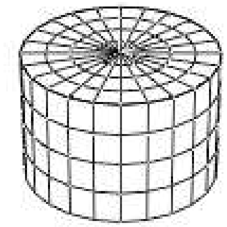
a. 11
b. 32
c. 46
d. 50
e. 92

Work Out: (Points indicated. Part credit possible. Show all work.)

10. (15 points) A rectangle sits on the xy-plane with its top 4 vertices on the elliptic paraboloid $\frac{x^2}{4} + \frac{y^2}{9} + z = 6$. Find the dimensions and volume of the largest such box.



11. (25 points) Verify Gauss' Theorem $\iiint_V \vec{\nabla} \cdot \vec{F}\, dV = \iint_{\partial V} \vec{F} \cdot d\vec{S}$



for the vector field $\vec{F} = (xz^2, yz^2, z(x^2+y^2))$ and the solid cylinder $x^2+y^2 \le 4$ for $-3 \le z \le 3$.

Be careful with orientations. Use the following steps:

First the Left Hand Side:

a. Compute the divergence:

$\vec{\nabla} \cdot \vec{F} =$

b. Express the divergence and the volume element in the appropriate coordinate system:

$\vec{\nabla} \cdot \vec{F} =$ $\qquad\qquad$ $dV =$

c. Compute the left hand side:

$\iiint_V \vec{\nabla} \cdot \vec{F}\, dV =$

Second the Right Hand Side:

The boundary surface consists of the cylindrical sides C, a disk T at the top and a disk B at the bottom with appropriate orientations.

d. Parametrize the cylinder C:

$\vec{R}(\theta, z) = \Big(\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\Big)$

e. Compute the tangent vectors:

$\vec{e}_\theta = \Big(\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\Big)$

$\vec{e}_z = \Big(\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\Big)$

f. Compute the normal vector:

$\vec{N} =$

g. Evaluate $\vec{F} = (xz^2, yz^2, z(x^2+y^2))$ on the cylinder:

$\vec{F}\Big|_{\vec{R}(r,\theta)} =$

h. Compute the dot product:

$\vec{F} \cdot \vec{N} =$

i. Compute the flux through C:

$\iint_C \vec{F} \cdot d\vec{S} =$

j. Parametrize the **TOP** disk T:

$\vec{R}(r,\theta) = \big(\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\big)$

k. Compute the tangent vectors:

$\vec{e}_r = \big(\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\big)$

$\vec{e}_\theta = \big(\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\big)$

l. Compute the normal vector:

$\vec{N} =$

m. Evaluate $\vec{F} = (xz^2, yz^2, z(x^2+y^2))$ on the top disk:

$\vec{F}\Big|_{\vec{R}(\theta,\varphi)} =$

n. Compute the dot product:

$\vec{F} \cdot \vec{N} =$

o. Compute the flux through T:

$\iint\limits_T \vec{F} \cdot d\vec{S} =$

p. Parametrize the **BOTTOM** disk B:

$\vec{R}(r,\theta) = \big(\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\big)$

q. Compute the tangent vectors:

$\vec{e}_r = \big(\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\big)$

$\vec{e}_\theta = \big(\_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_\_\_\_\_\big)$

r. Compute the normal vector:

$\vec{N} =$

s. Evaluate $\vec{F} = (xz^2, yz^2, z(x^2+y^2))$ on the bottom disk:

$\vec{F}\Big|_{\vec{R}(\theta,\varphi)} =$

t. Compute the dot product:

$\vec{F} \cdot \vec{N} =$

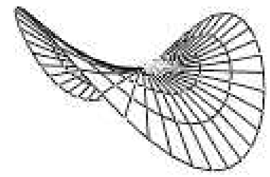
u. Compute the flux through B:

$\iint\limits_B \vec{F} \cdot d\vec{S} =$

v. Compute the **TOTAL** right hand side:

$\iint\limits_{\partial V} \vec{F} \cdot d\vec{S} =$

12. (15 points) Compute $\iint \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ for $\vec{F} = (-y, x, z)$ over the hyperbolic paraboloid $z = x^2 - y^2$ parametrized by



$$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2(\cos^2\theta - \sin^2\theta))$$

for $r \le 2$ oriented upward.

HINTS: Use Stokes Theorem.

What is the value of r on the boundary?