

Name \_\_\_\_\_

MATH 221 Exam 3 Version A Fall 2019

Section 504 P. Yasskin

1-9	/54	11	/25
10	/25	Total	/104

Multiple Choice: (6 points each. No part credit.)

1. Compute the integral  $\iint x^2 y dA$  over the region between  $y = 4x^2$  and  $y = x^4$  in the first quadrant.

a.  $\frac{2^{12}}{77}$

b.  $\frac{2^{13}}{77}$

c.  $\frac{2^6}{35}$

d.  $\frac{2^7}{35}$

e.  $\frac{2^8}{35}$

2. Find the average value of the function  $f(x,y) = x^2 y$  on the rectangle  $[0, 3] \times [0, 4]$ .

a.  $\frac{3}{2}$

b. 6

c. 12

d. 18

e. 72

3. Compute  $\int_0^4 \int_{\sqrt{y}}^2 e^{x^3} dx dy$

a.  $\frac{e^8}{3} - \frac{1}{3}$

b.  $\frac{e^{16}}{4} - \frac{1}{4}$

c.  $\frac{e^{16}}{4} - \frac{e^4}{4}$

d.  $\frac{e^{64}}{4} - \frac{1}{4}$

e.  $\frac{e^{64}}{4} - \frac{e^{16}}{4}$

4. Find the area of the **inner loop** of the limaçon  $r = 4 \cos \theta - 2$ .  
HINT: Find the angles at which  $r = 0$ .

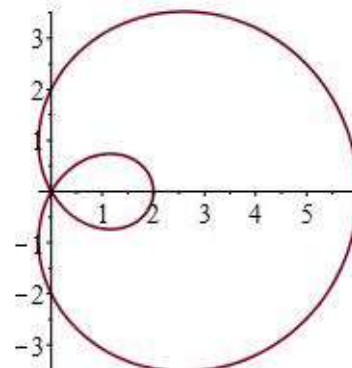
a.  $2\pi + 6\sqrt{3}$

b.  $4\pi - 6\sqrt{3}$

c.  $6\pi - 4\sqrt{3}$

d.  $8\pi + 4\sqrt{3}$

e.  $8\pi + 6\sqrt{3}$



5. Find the mass of the solid between the paraboloids  $z = x^2 + y^2$  and  $z = 8 - x^2 - y^2$  if the density is  $\delta = z$ .
- $4\pi$
  - $32\pi$
  - $36\pi$
  - $64\pi$
  - $120\pi$
6. Find the volume of the solid above the cone  $z = \sqrt{x^2 + y^2}$  below the sphere  $x^2 + y^2 + z^2 = 9$ .
- $9\pi$
  - $18\pi$
  - $9\pi\sqrt{2}$
  - $9\pi\left(1 - \frac{\sqrt{3}}{2}\right)$
  - $18\pi\left(1 - \frac{1}{\sqrt{2}}\right)$
7. Compute the circulation of the vector field  $\vec{F} = \langle -yz, xz, z^2 \rangle$  counterclockwise around the circle  $\vec{r}(\theta) = (3 \cos \theta, 3 \sin \theta, 2)$ . Note:  $Circ = \oint \vec{F} \cdot d\vec{s}$
- $4\pi$
  - $9\pi$
  - $36\pi$
  - $54\pi$
  - $72\pi$

8. Find the  $x$ -component of the center of mass of the twisted helix  $\vec{r}(t) = \left(t, t^2, \frac{2}{3}t^3\right)$  for  $0 \leq t \leq 3$ , if the density is  $\delta = 2$ .
- a.  $\bar{x} = 42$
  - b.  $\bar{x} = 90$
  - c.  $\bar{x} = \frac{15}{14}$
  - d.  $\bar{x} = \frac{14}{15}$
  - e.  $\bar{x} = \frac{15}{7}$

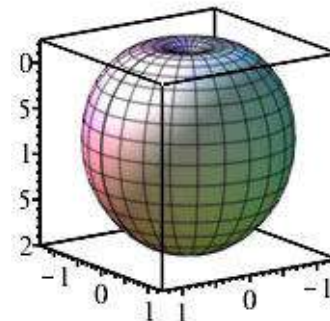
9. Compute  $\iiint \vec{\nabla} \cdot \vec{F} dV$  over the solid between the spheres  $x^2 + y^2 + z^2 = 4$  and  $x^2 + y^2 + z^2 = 16$  if  $\vec{F} = \langle xz^2, yz^2, z^3 \rangle$ .
- a.  $\frac{4^6\pi}{3}$
  - b.  $\frac{2^7\pi}{3}$
  - c.  $\frac{4\pi}{3}(4^5 - 2^5)$
  - d.  $\frac{2\pi}{3}(4^5 - 2^5)$
  - e.  $\frac{2\pi}{3}(4^6 - 2^6)$

Work Out: (Points indicated. Part credit possible. Show all work.)

10. (25 points) The apple at the right is given in spherical coordinates by  $\rho = 1 - \cos \varphi$ .

HINTS:  $(1 - u)^3 = 1 - 3u + 3u^2 - u^3$

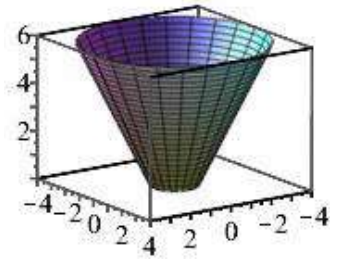
$$(1 - u)^4 = 1 - 4u + 6u^2 - 4u^3 + u^4$$



- a. Find the volume.

- b. Find the centroid. (Part credit: Set up the integrals and say what you do with the answers.)

11. (25 points) Consider a bowl given in cylindrical coordinates by  $z = 2r - 2$  for  $1 \leq r \leq 4$  oriented **down and out**. The surface may be parametrized by  $\vec{R}(r, \theta) = (r \cos \theta, r \sin \theta, 2r - 2)$ .



a. Find the surface area.

i. Find the tangent vectors:

$$\vec{e}_r =$$

$$\vec{e}_\theta =$$

ii. Find the normal:

$$\vec{N} =$$

iii. Find the length of the normal:

$$|\vec{N}| =$$

iv. Find the area:

$$A =$$

b. Find the flux of the vector field  $\vec{F} = \langle x, y, -z \rangle$  **down** through the surface.

i. Evaluate the vector field on the surface:

$$\vec{F}|_{\vec{R}} =$$

ii. Restate the normal:

$$\vec{N} =$$

iii. Compute the flux:

$$Flux = \iint \vec{F} \cdot d\vec{S} =$$