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MATH 221 Exam 3 Version A Fall 2019

Section 505 P. Yasskin

1-9	/54	11	/25
10	/25	Total	/104

Multiple Choice: (6 points each. No part credit.)

1. Compute the integral $\iint x^2 y\,dA$ over the region between $y = 4x^2$ and $y = x^4$ in the first quadrant.

a. $\frac{2^{12}}{77}$

b. $\frac{2^{13}}{77}$

c. $\frac{2^{6}}{35}$

d. $\frac{2^{7}}{35}$

e. $\frac{2^{8}}{35}$

2. Find the average value of the function $f(x,y) = x^2 y$ on the rectangle $[0,3] \times [0,4]$.

a. $\frac{3}{2}$

b. 6

c. 12

d. 18

e. 72

3. Compute $\int_0^4 \int_{\sqrt{y}}^2 e^{x^3}\,dx\,dy$

a. $\frac{e^8}{3} - \frac{1}{3}$

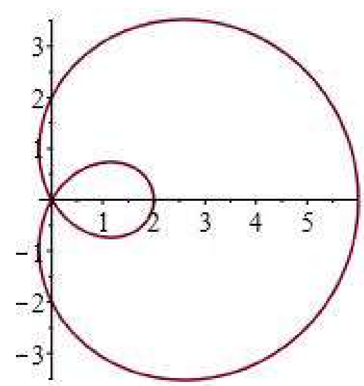
b. $\frac{e^{16}}{4} - \frac{1}{4}$

c. $\frac{e^{16}}{4} - \frac{e^4}{4}$

d. $\frac{e^{64}}{4} - \frac{1}{4}$

e. $\frac{e^{64}}{4} - \frac{e^{16}}{4}$

4. Find the area of the **inner loop** of the limacon $r = 4\cos\theta - 2$.
HINT: Find the angles at which $r = 0$.



a. $2\pi + 6\sqrt{3}$

b. $4\pi - 6\sqrt{3}$

c. $6\pi - 4\sqrt{3}$

d. $8\pi + 4\sqrt{3}$

e. $8\pi + 6\sqrt{3}$

5. Find the mass of the solid between the paraboloids $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$ if the density is $\delta = z$.

a. 4π

b. 32π

c. 36π

d. 64π

e. 120π

6. Find the volume of the solid above the cone $z = \sqrt{x^2 + y^2}$ below the sphere $x^2 + y^2 + z^2 = 9$.

a. 9π

b. 18π

c. $9\pi\sqrt{2}$

d. $9\pi\left(1 - \frac{\sqrt{3}}{2}\right)$

e. $18\pi\left(1 - \frac{1}{\sqrt{2}}\right)$

7. Compute the circulation of the vector field $\vec{F} = \langle -yz, xz, z^2 \rangle$ counterclockwise around the circle $\vec{r}(\theta) = (3\cos\theta, 3\sin\theta, 2)$. Note: $Circ = \oint \vec{F} \cdot d\vec{s}$

a. 4π

b. 9π

c. 36π

d. 54π

e. 72π

8. Find the x-component of the center of mass of the twisted helix $\vec{r}(t) = \left(t, t^2, \frac{2}{3}t^3\right)$ for $0 \le t \le 3$, if the density is $\delta = 2$.

a. $\bar{x} = 42$

b. $\bar{x} = 90$

c. $\bar{x} = \frac{15}{14}$

d. $\bar{x} = \frac{14}{15}$

e. $\bar{x} = \frac{15}{7}$

9. Compute $\iiint \vec{\nabla} \cdot \vec{F}\, dV$ over the solid between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 16$ if $\vec{F} = \langle xz^2, yz^2, z^3 \rangle$.

a. $\frac{4^6 \pi}{3}$

b. $\frac{2^7 \pi}{3}$

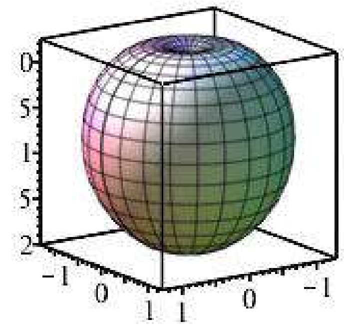
c. $\frac{4\pi}{3}(4^5 - 2^5)$

d. $\frac{2\pi}{3}(4^5 - 2^5)$

e. $\frac{2\pi}{3}(4^6 - 2^6)$

Work Out: (Points indicated. Part credit possible. Show all work.)

10. (25 points) The apple at the right is given in spherical coordinates by $\rho = 1 - \cos\varphi$.



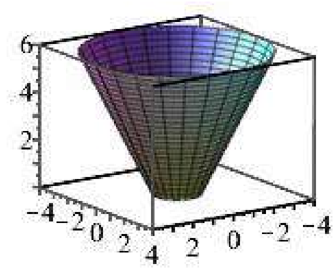
HINTS: $(1-u)^3 = 1 - 3u + 3u^2 - u^3$

$(1-u)^4 = 1 - 4u + 6u^2 - 4u^3 + u^4$

a. Find the volume.

b. Find the centroid. (Part credit: Set up the integrals and say what you do with the answers.)

11. (25 points) Consider a bowl given in cylindrical coordinates by $z = 2r - 2$ for $1 \le r \le 4$ oriented **down and out**. The surface may be parametrized by $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, 2r-2)$.



a. Find the surface area.

i. Find the tangent vectors:

$\vec{e}_r =$

$\vec{e}_\theta =$

ii. Find the normal:

$\vec{N} =$

iii. Find the length of the normal:

$|\vec{N}| =$

iv. Find the area:

$A =$

b. Find the flux of the vector field $\vec{F} = \langle x, y, -z \rangle$ **down** through the surface.

i. Evaluate the vector field on the surface:

$\vec{F}\big|_{\vec{R}} =$

ii. Restate the normal:

$\vec{N} =$

iii. Compute the flux:

$Flux = \iint \vec{F} \cdot d\vec{S} =$