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MATH 221 Final Exam Version B Fall 2019

Section 505 P. Yasskin

1-11	/55	13	/10
12	/15	14	/25
		Total	/105

Multiple Choice: (5 points each. No part credit.)

1. A triangle has vertices $A = (3,2,1)$, $B = (3,3,2)$ and $C = (4,4,2)$. Find the angle at A.

a. $0°$

b. $30°$

c. $45°$

d. $60°$

e. $90°$

2. Find the area of the triangle with vertices $A = (3,2,1)$, $B = (3,3,2)$ and $C = (4,4,2)$.

a. $2\sqrt{6}$

b. $\sqrt{6}$

c. $\dfrac{\sqrt{6}}{2}$

d. $\sqrt{3}$

e. $\dfrac{\sqrt{3}}{2}$

3. Find the arc length of the curve $\vec{r}(t) = (t^2, 2t, \ln t)$ between $(1,2,0)$ and $(e^2, 2e, 1)$.

a. $e^2 + 1$
b. $e^2 - 1$
c. e^2
d. $\sqrt{e^2 + 1}$
e. $\sqrt{e^2 + 1} - 1$

4. Find the average value of the function $f(x,y,z) = x$ along the curve $\vec{r}(t) = (t^2, 2t, \ln t)$ between $(1,2,0)$ and $(e^2, 2e, 1)$.

a. $\frac{e^2}{2} - \frac{1}{e^2}$
b. $\frac{e^4}{2} - \frac{e^2}{2} - 1$
c. $\frac{e^2}{2} - \frac{1}{2} - \frac{1}{e^2}$
d. $\frac{e^4}{2} + \frac{e^2}{2} - 1$
e. $\frac{e^2}{2} + \frac{1}{2} - \frac{1}{e^2}$

5. Find the plane tangent to $x\sin y + 2z\cos y = 2$ at $(x,y,z) = \left(\sqrt{2}, \frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$. The z-intercept is:

a. 2
b. $\sqrt{2}$
c. $\frac{1}{\sqrt{2}}$
d. $\frac{1}{2\sqrt{2}}$
e. 0

6. The velocity field of the water in a sink is $\vec{V} = \langle -x^2 y, xy^2 \rangle$ Find the circulation of the water, $Circ = \oint \vec{V} \cdot d\vec{s}$, counterclockwise around the circle $x^2 + y^2 = 4$.

HINT: Use Green's Theorem.

a. 2π
b. $\frac{16}{3}\pi$
c. 8π
d. 32π
e. 64π

7. Compute $\int \vec{F} \cdot d\vec{s}$ for $\vec{F} = \langle 4x^3, 3y^2, 2z \rangle$ along the curve $\vec{r}(t) = \left(t\sin\left(\frac{\pi}{2}t\right), 2^t \cos\left(\frac{\pi}{2}t\right), 2^t \sin\left(\frac{\pi}{2}t\right) \right)$ from $t = 0$ to $t = 1$.
HINT: Find a scalar potential.

a. 0
b. 1
c. 2
d. 3
e. 4

8. Compute $\iint_S \vec{\nabla} \times \vec{G} \cdot d\vec{S}$ for $\vec{G} = \langle y, -x, z \rangle$ over the surface $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, 16 - r^4)$ for $0 \le r \le 2$ and $0 \le \theta \le 2\pi$ oriented up and out.
HINT: Use a theorem.

a. -8π
b. -4π
c. -2π
d. 2π
e. 4π

9. Find the centroid of the solid hemisphere $0 \le z \le \sqrt{4 - x^2 - y^2}$.

a. $\left(0, 0, \frac{1}{2}\right)$

b. $\left(0, 0, \frac{3}{4}\right)$

c. $\left(0, 0, \frac{4}{3}\right)$

d. $(0, 0, 4\pi)$

e. $(0, 0, 2\pi)$

10. Find the area of the surface $\vec{R}(u,v) = (u, v, u + v)$ for $0 \le u \le 2$ and $0 \le v \le 3$.

a. $12\sqrt{2}$

b. $12\sqrt{3}$

c. $6\sqrt{2}$

d. $6\sqrt{3}$

e. $3\sqrt{2}$

11. Find the flux of $\vec{F} = \langle y, -z, x\rangle$ upward thru the surface $\vec{R}(u,v) = (u, v, u + v)$ for $0 \le u \le 2$ and $0 \le v \le 3$.

a. 30

b. 24

c. 18

d. 12

e. 0

Work Out: (Points indicated. Part credit possible. Show all work.)

12. (15 points) The Ideal Gas Law says the Pressure, P, Density, δ, and Temperature, T, are related by $P = k\delta T$ for some constant k. We will assume the atmosphere is an ideal gas with $k = 2$. A weather baloon measures the Density and Temperature to be

$$\delta = 0.1\ \frac{\text{gm}}{\text{m}^3} \qquad T = 270°\text{K}$$

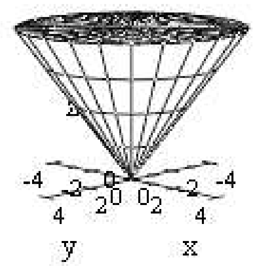
and their gradients to be

$$\vec{\nabla}\delta = \langle 0.01, 0.02, 0.03 \rangle \qquad \vec{\nabla}T = \langle 3, 1, -1 \rangle$$

Find the gradient of the Pressure.

13. (10 points) Find the largest value of $f = xy^2z^3$ on the plane $2x + 3y + 6z = 72$.

14. (25 points) Verify Gauss' Theorem $\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV = \iint\limits_{\partial V} \vec{F} \cdot d\vec{S}$



for the vector field $\vec{F} = \left(xz, yz, 3z\sqrt{x^2 + y^2} \right)$ and the solid cone $\sqrt{x^2 + y^2} \le z \le 2$.

Be careful with orientations. Use the following steps:

First the Left Hand Side:

a. Compute the divergence:

$\vec{\nabla} \cdot \vec{F} =$

b. Express the divergence and the volume element in the appropriate coordinate system:

$\vec{\nabla} \cdot \vec{F} =$ $\qquad\qquad$ $dV =$

c. Compute the left hand side:

$\iiint\limits_V \vec{\nabla} \cdot \vec{F}\, dV =$

Second the Right Hand Side:

The boundary surface consists of a hemisphere H and a disk D with appropriate orientations.

The disk D may be parametrized as: $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, 2)$

d. Compute the tangent vectors:

$\vec{e}_r =$

$\vec{e}_\theta =$

e. Compute the normal vector:

$\vec{N} =$

f. Evaluate $\vec{F} = \left(xz, yz, 3z\sqrt{x^2+y^2}\right)$ on the disk:

$\vec{F}\Big|_{\vec{R}(r,\theta)} =$

g. Compute the dot product:

$\vec{F} \bullet \vec{N} =$

h. Compute the flux through D:

$\iint\limits_{D} \vec{F} \bullet d\vec{S} =$

The cone C may be parametrized as: $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r)$

i. Compute the tangent vectors:

$\vec{e}_r =$

$\vec{e}_\theta =$

j. Compute the normal vector:

$\vec{N} =$

k. Evaluate $\vec{F} = \left(xz, yz, 3z\sqrt{x^2+y^2}\right)$ on the cone:

$\vec{F}\Big|_{\vec{R}(r,\theta)} =$

l. Compute the dot product:

$\vec{F} \bullet \vec{N} =$

m. Compute the flux through C:

$\iint\limits_{C} \vec{F} \bullet d\vec{S} =$

n. Compute the **TOTAL** right hand side: