Name________________________UIN__________________

MATH 221 Exam 1 Fall 2021

Sections 504/505 P. Yasskin

1-10	/50	12	/10
11	/10	13	/35
		Total	/105

Multiple Choice: (5 points each. No part credit.)

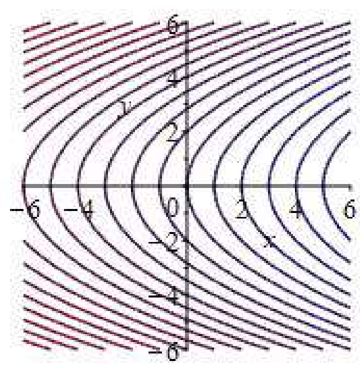
1. A point is given in cylindrical coordinates by $(r,\theta,z) = \left(3, \frac{\pi}{3}, 3\right)$.
Find its spherical coordinates.

$$(\rho,\varphi,\theta) = (\_\_\_\_\_\_,\_\_\_\_\_\_,\_\_\_\_\_\_)$$

2. A sphere is centered at $(1,3,5)$ and is tangent to the plane $y = 1$.
What is the equation of the sphere?

a. $(x-1)^2 + (y-3)^2 + (z-5)^2 = 0$
b. $(x-1)^2 + (y-3)^2 + (z-5)^2 = 1$
c. $(x-1)^2 + (y-3)^2 + (z-5)^2 = 2$
d. $(x-1)^2 + (y-3)^2 + (z-5)^2 = 3$
e. $(x-1)^2 + (y-3)^2 + (z-5)^2 = 4$
f. $(x-1)^2 + (y-3)^2 + (z-5)^2 = 5$
g. $(x-1)^2 + (y-3)^2 + (z-5)^2 = 6$
h. $(x-1)^2 + (y-3)^2 + (z-5)^2 = 7$
i. $(x-1)^2 + (y-3)^2 + (z-5)^2 = 8$
j. $(x-1)^2 + (y-3)^2 + (z-5)^2 = 9$

3. This is the contour plot of which function?



a. $f(x,y) = y - \frac{x^2}{4}$

b. $f(x,y) = y + \frac{x^2}{4}$

c. $f(x,y) = x - \frac{y^2}{4}$

d. $f(x,y) = x + \frac{y^2}{4}$

4. Write $\langle 5,5,5\rangle$ as a linear combination of $\langle 3,-1,2\rangle$ and $\langle 1,3,2\rangle$ or type "impossible" in both boxes.

$$\langle 5,5,5\rangle = \_\_\_\_\_\_\langle 3,-1,2\rangle + \_\_\_\_\_\_\langle 1,3,2\rangle$$

5. Find the angle between the vectors $\langle 2,-2,1\rangle$ and $\langle 1,-4,-1\rangle$.

a. $0°$
b. $30°$
c. $45°$
d. $60°$
e. $90°$
f. $120°$
g. $135°$
h. $150°$
i. $180°$

6. Write $\vec{v} = \langle 2, -8, -2 \rangle$ as the sum of two vectors $\vec{p}$ and $\vec{q}$ where $\vec{p}$ is parallel to $\vec{u} = \langle 2, -2, 1 \rangle$ and $\vec{q}$ is perpendicular to $\vec{u}$.

$$\vec{v} = \langle 2, -8, -2 \rangle = \vec{p} + \vec{q}$$

where

$$\vec{p} = \langle \_\_\_\_, \_\_\_\_, \_\_\_\_ \rangle \quad \text{and} \quad \vec{q} = \langle \_\_\_\_, \_\_\_\_, \_\_\_\_ \rangle$$

7. If $\vec{a}$ points DOWN and $\vec{b}$ points SOUTHWEST, in what direction does $\vec{a} \times \vec{b}$ point?

a. NORTH
b. SOUTH
c. EAST
d. WEST
e. DOWN
f. NORTHEAST
g. NORTHWEST
h. SOUTHEAST
i. SOUTHWEST
j. UP

8. Find the volume of the parallelepiped with edge vectors

$$\vec{p} = \langle 2, 1, 3 \rangle \qquad \vec{q} = \langle 3, 2, 0 \rangle \qquad \vec{r} = \langle 4, 0, 1 \rangle$$

$$V = \_\_\_\_\_\_\_$$

9. Find the standard equation of the plane which passes through the point $P = (3, 2, 1)$ and is perpendicular to the line $\vec{r}(t) = (2 + t, 3 - 2t, 1 + 4t)$.

$$\_\_\_\_\_\_ x + \_\_\_\_\_\_ y + \_\_\_\_\_\_ z = \_\_\_\_\_\_$$

10. Identify the surface

$$9x^2 - 36x - 4y^2 + 8y + z^2 + 4z + 36 = 0$$

a. Sphere
b. Ellipsoid
c. Hyperboloid of 1 sheet
d. Hyperboloid of 2 sheets
e. Cone
f. Elliptic Paraboloid
g. Hyperbolic Paraboloid
h. Elliptic Cylinder
i. Hyperbolic Cylinder
j. Parabolic Cylinder

Work Out: (Points indicated. Part credit possible. Show all work.)

11. (10 points) Find the point of intersection of the line

$$\frac{x+2}{2} = \frac{y+5}{3} = \frac{z+5}{4}$$

and the plane:

$$x - y + z = 4$$

(You will be graded on your work.)

12. (10 points) Consider the two planes

$$P_1 : \qquad x + y + z = 3$$

$$P_2 : \qquad x - y + 2z = 1$$

Compute each of the following quantities. (You will be graded on your work.)

a. The normal vectors to the planes:

$\vec{N}_1 =$ ____________________

$\vec{N}_2 =$ ____________________

b. The direction of the line of intersection:

$\vec{v} =$ ____________________

c. A point on the line of intersection:

$P =$ ____________________

d. The equation of the line of intersection:

$\vec{r}(t) =$ ____________________

13. (35 points) Consider the parametric curve $\vec{r}(t) = \left(t^2, \frac{2}{3}t^3, \frac{1}{4}t^4\right)$.
Compute each of the following quantities. (You will be graded on your work.)

a. Velocity $\vec{v} =$

b. Acceleration $\vec{a} =$

c. Speed $|\vec{v}| =$

d. Unit Tangent Vector $\hat{T} =$

e. Arc Length between $A = (0,0,0)$ and $B = \left(4, \frac{16}{3}, 4\right)$
$L =$

f. The Scalar Line Integral of $f(x,y,z) = x$ between $A = (0,0,0)$ and $B = \left(4, \frac{16}{3}, 4\right)$
$\int_A^B f\,ds =$

g. The Vector Line Integral of $\vec{F}(x,y,z) = \langle 4z, 3y, 2x \rangle$ between $A = (0,0,0)$ and $B = \left(4, \frac{16}{3}, 4\right)$
$\int_A^B \vec{F} \cdot d\vec{s} =$