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1-13	/65	15	/20
14	/15	16	/10
		Total	/110

MATH 221 Final Exam 505 Fall 2021

Sections 504/505 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Find the equation of the plane tangent to the graph of $z = f(x,y) = x^2y + xy^2$ at $(2,1)$. Its z-intercept is

a. -6 f. 6

b. -10 g. 10

c. -12 h. 12

d. -16 i. 16

e. -24 j. 24

2. Find the equation of the line perpendicular to the hyperboloid $xyz = 6$ at the point $(1,2,3)$.

a. $x + 2y + 3z = 14$ f. $6x + 3y + 2z = 18$

b. $x + 2y + 3z = 18$ g. $6x + 3y + 2z = 49$

c. $(x,y,z) = (1 + 6t, 2 - 3t, 3 + 2t)$ h. $(x,y,z) = (1 + 6t, 2 + 3t, 3 + 2t)$

d. $(x,y,z) = (6 + t, 3 - 2t, 2 + 3t)$ i. $(x,y,z) = (6 + t, 3 + 2t, 2 + 3t)$

e. $(x,y,z) = (1 + 2t, 2 - 3t, 3 + 6t)$ j. $(x,y,z) = (1 + 2t, 2 + 3t, 3 + 6t)$

3. Find the area of the triangle with vertices $(1,1,1)$, $(2,2,4)$ and $(1,3,9)$.

a. 1
b. 2
c. 3
d. 4
e. 6
f. $\sqrt{2}$
g. $2\sqrt{2}$
h. $3\sqrt{2}$
i. $4\sqrt{2}$
j. $6\sqrt{2}$

4. Find the point where the line $(x,y,z) = (3,2,1) + t(1,2,3)$ intersects the plane $x - y + z = -2$. At this point, $x + y + z =$

a. -6
b. -4
c. -2
d. -1
e. 0
f. 6
g. 4
h. 2
i. 1
j. none of these

5. A particle moves along the curve $\vec{r}(t) = (t,t^2,t^3)$ from $(1,1,1)$ to $(2,4,8)$ due to the force $\vec{F} = (z,y,x)$. Find the work done by the force.

a. 14
b. 24
c. 36
d. 45
e. 48
f. $\frac{45}{2}$
g. $\frac{70}{3}$
h. $\frac{93}{5}$
i. $\frac{96}{5}$
j. $\frac{186}{5}$

6. A circuit has two resistors $R_1 = 200\ \Omega$ and $R_2 = 300\ \Omega$ in parallel.
The net resistance R satisfies $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. If R_1 is increasing at $2\ \Omega$/sec and R_2 is decreasing at $9\ \Omega$/sec at what rate is R changing?

a. $-\frac{9}{50}\ \Omega$/sec
b. $-\frac{18}{25}\ \Omega$/sec
c. $-\frac{9}{25}\ \Omega$/sec
d. $-\frac{36}{25}\ \Omega$/sec
e. $-500\ \Omega$/sec
f. $\frac{9}{50}\ \Omega$/sec
g. $\frac{18}{25}\ \Omega$/sec
h. $\frac{9}{25}\ \Omega$/sec
i. $\frac{36}{25}\ \Omega$/sec
j. $500\ \Omega$/sec

7. Compute $\iint x^2\, dA$ over the region between the parabolas $y = 4 - x^2$ and $y = 8 - 2x^2$.

a. $\frac{1}{15}$
b. $\frac{2}{15}$
c. $\frac{4}{15}$
d. $\frac{8}{15}$
e. $\frac{16}{15}$
f. $\frac{32}{15}$
g. $\frac{64}{15}$
h. $\frac{128}{15}$
i. $\frac{256}{15}$
j. $\frac{512}{15}$

8. Compute $\iint_R \frac{1}{x^2+y^2}\,dx\,dy$ over the ring $9 \le x^2+y^2 \le 16$.

a. $2\pi \ln \frac{16}{9}$

b. $4\pi \ln \frac{16}{9}$

c. $\pi \ln \frac{4}{3}$

d. $2\pi \ln \frac{4}{3}$

e. $4\pi \ln \frac{4}{3}$

f. $2\pi \ln \frac{9}{16}$

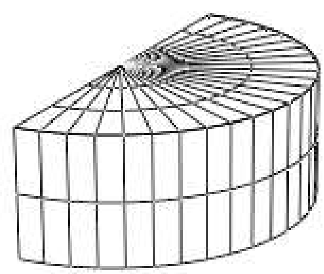
g. $4\pi \ln \frac{9}{16}$

h. $\pi \ln \frac{3}{4}$

i. $2\pi \ln \frac{3}{4}$

j. $4\pi \ln \frac{3}{4}$

9. Compute $\iiint y\,dV$ over the half cylinder $x^2+y^2 \le 9$ with $y \ge 0$ for $-1 \le z \le 1$.



a. $\frac{3\pi}{2}$

b. $\frac{9\pi}{2}$

c. 9π

d. 18π

e. 36π

f. $\frac{3}{2}$

g. $\frac{9}{2}$

h. 9

i. 18

j. 36

10. Compute $\int_{(1,1,1)}^{(1/e,1/e,e^2)} \vec{F} \cdot d\vec{s}$ where $\vec{F} = (z, z, x+y)$ along the curve $\vec{r}(t) = (e^{-t}, e^{-t}, e^{2t})$.

HINT: Find a scalar potential.

a. $4-4e$
b. $4-2e$
c. $2-2e$
d. $2-4e$
e. $4+4e$
f. $4e-4$
g. $2e-4$
h. $2e-2$
i. $4e-2$
j. $2+2e$

11. Compute $\oint (\sin(x^3) - 2x^2y)\,dx + (2xy^2 + \cos(y^3))\,dy$ counterclockwise around the circle $x^2+y^2 = 9$.

HINT: Use a theorem.

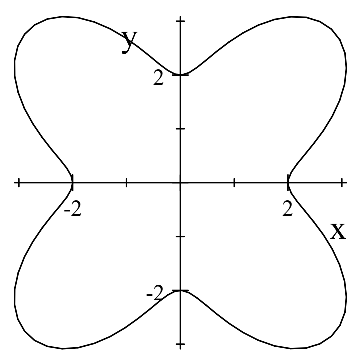
a. 3π
b. 6π
c. 9π
d. 12π
e. 18π
f. 27π
g. 36π
h. 54π
i. 72π
j. 81π

12. Compute $\iint_{\partial C} \vec{F} \cdot d\vec{S}$ for $\vec{F} = (x^3 z, y^3 z, x^2 + y^2)$ over the total surface of the cylinder $x^2 + y^2 \le 4$ for $0 \le z \le 3$.

HINT: Use Gauss' Theorem.

a. 6π f. 36π
b. 9π g. 54π
c. 12π h. 108π
d. 18π i. 216π
e. 27π j. 432π

13. Compute $\oint \vec{\nabla} f \cdot d\vec{s}$ counterclockwise once around the polar curve $r = 3 - \cos(4\theta)$ for the function $f(x,y) = x^2 + y$.



HINT: Use the FTCC or Green's Theorem.

a. π **f**. 8π
b. 2π **g**. 12π
c. 4π **h**. 16π
d. 6π **i**. 32π
e. 0 **j**. none of these

Work Out: (Points indicated. Part credit possible. Show all work.)

14. (15 points) We want to make a cylindrical aluminum can with lids to hold 32π cm^3.

However, the top and bottom are to be twice as thick as the sides.

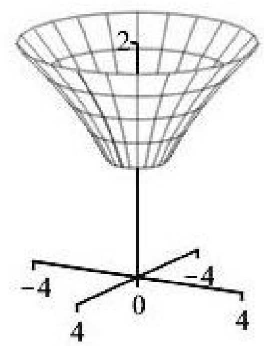
The metal for the sides is the surface area, $2\pi rh$.

The metal for each of the 2 ends is twice their area, πr^2.

So the total amount of metal is: $M = 2\pi rh + 4\pi r^2$.

Find the radius and height of the can and the amount of metal needed to make a can which uses the least amount of metal.

15. (20 points) Verify Stokes' Theorem $\iint_S \vec{\nabla} \times \vec{F} \bullet d\vec{S} = \oint_{\partial S} \vec{F} \bullet d\vec{s}$



for the vector field $\vec{F} = \langle -yz, xz, z^2 \rangle$ and the funnel $r = z^2$ for $1 \le z \le 2$ oriented down and out.

Be careful with orientations. Use the following steps:

First the Left Hand Side:

a. Compute the curl $\vec{\nabla} \times \vec{F}$ in rectangular coordinates.

$\vec{\nabla} \times \vec{F} =$

b. The funnel surface, S, may be parametrized by $\vec{R}(z,\theta) = (z^2 \cos\theta, z^2 \sin\theta, z)$.

What is $\vec{\nabla} \times \vec{F}$ on the funnel?

$\vec{\nabla} \times \vec{F}\Big|_{\vec{R}(z,\theta)} =$

c. Find the normal to the funnel.

$\vec{e}_z =$

$\vec{e}_\theta =$

$\vec{N} =$

d. Compute the integral $\iint_P \vec{\nabla} \times \vec{F} \bullet d\vec{S}$.

$\vec{\nabla} \times \vec{F} \bullet \vec{N} =$

$\iint_S \vec{\nabla} \times \vec{F} \bullet d\vec{S} =$

Second the Right Hand Side:

e. The circle, T, at the top of the funnel may be parametrized by $\vec{r}(\theta) = (4\cos\theta, 4\sin\theta, 2)$.

What is $\vec{F} = \langle -yz, xz, z^2 \rangle$ on the top circle?

$\vec{F} =$

f. What is the tangent vector to the circle?

$\vec{v} =$

g. Compute the integral $\oint_T \vec{F} \cdot d\vec{s}$.

$\vec{F} \cdot \vec{v} =$

$\int_T \vec{F} \cdot d\vec{s} =$

h. The circle, B, at the bottom of the funnel may be parametrized by $\vec{r}(\theta) = (\cos\theta, \sin\theta, 1)$.

What is $\vec{F} = \langle -yz, xz, z^2 \rangle$ on the bottom circle?

$\vec{F} =$

i. What is the tangent vector to the circle?

$\vec{v} =$

j. Compute the integral $\oint_B \vec{F} \cdot d\vec{s}$.

$\vec{F} \cdot \vec{v} =$

$\int_B \vec{F} \cdot d\vec{s}$

k. Compute the **TOTAL** right hand side:

$\int_{\partial S} \vec{F} \cdot d\vec{S} =$

16. (10 points) Find the volume and z-component of the centroid of the solid between the surfaces

$$z = (x^2 + y^2)^{3/2} \quad \text{and} \quad z = 8.$$