Name________________________

MATH 221 Exam 2 Spring 2023

Section 501 P. Yasskin

Multiple Choice: (6 points each. No part credit.)

1-9	/54	12	/14
10	/12	13	/12
11	/12	Total	/104

1. The volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Its radius is measured to be $r = 2 \pm .02$ cm and its height is measured to be $h = 6 \pm .03$ cm.
Using the linear approximation, we compute $V = 8\pi \pm \Delta V$ where $\Delta V =$

a. 0.6π

b. 0.4π

c. 0.3π

d. 0.2π

e. 0.1π

2. The function $f = xy + \frac{3}{x} - \frac{9}{y}$ has a critical point at $(x,y) = (-1,3)$.
Use the Second Derivative Test to classify this critical point.

a. Local Minimum

b. Local Maximum

c. Inflection Point

d. Saddle Point

e. Test Fails

3. Find the plane tangent to the graph of $z = xe^y$ at the point $(3, 0)$. Its z-intercept is

a. $-e$

b. -2

c. 0

d. 2

e. e

4. Find the plane tangent to the graph of $xz^3 + zy^2 + yx^4 = 8$ at the point $(1, 0, 2)$. Its z-intercept is

a. $\frac{1}{3}$

b. $\frac{2}{3}$

c. $\frac{4}{3}$

d. $\frac{8}{3}$

e. 32

5. Sidney says the Hessian of $f(x,y,z) = x\sin y + y\cos x$ is

$$\begin{pmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{pmatrix} = \begin{pmatrix} -y\cos x & \sin x - \cos y \\ \cos y - \sin x & -x\sin y \end{pmatrix}$$

Which entry is wrong?

a. f_{xx}

b. f_{yx}

c. f_{xy}

d. f_{yy}

e. None of them.

6. If $\vec{F} = (yz, -xz, z^2)$, compute $\vec{F} \bullet \vec{\nabla} \times \vec{F}$.

a. $-2z^3$

b. z^3

c. $z^3 + xyz$

d. $-2z^3 + 2xyz$

e. 0

7. Find the point (x,y) at which the divergence of $\vec{F} = \langle 6x^2 - xy^2, -y^2 - 2x^2y \rangle$ is a maximum.

a. $(3,1)$

b. $(-3,1)$

c. $(-3,-1)$

d. $(3,-1)$

e. $(0,0)$

8. Find the mass of a wire in the shape of the semi-circle $\vec{r}(\theta) = (4\cos\theta, 4\sin\theta)$ for $0 \le \theta \le \pi$ if the linear density is $\delta = y$.

a. 2π

b. 8π

c. 8

d. 16

e. 32

9. A bead is pushed along a wire in the shape of the twisted cubic $\vec{r}(t) = (t^3, t^2, t)$ by the force $\vec{F} = \langle z^3, yz^2, xz^2 \rangle$ from $(1,1,1)$ to $(8,4,2)$. Find the work done.

a. 186

b. $\frac{384}{7}$

c. $\frac{381}{7}$

d. 63

e. 64

Work Out: (Points indicated. Part credit possible. Show all work.)

10. $(12$ points$)$ Find the point $P=(x,y,z)$ on the plane $x+y-z=2$ which is closest to the point $Q=(1,0,2)$. Find the distance from P to Q.

$P=(\_\_\_,\_\_\_,\_\_\_)$

$D=$ __________

11. $(12$ points$)$ As Duke Skywater flies the Centurion Eagle through the galaxy he wants to maximize the Power of the Force which is given by $F=\frac{1}{D}$ where D is the dark matter density given by $D=x^3+y^3+z^3$.
If his current position is $\vec{r}=(2,1,1)$ and his current velocity is $\vec{v}=(0.5,-0.2,-0.8)$, what is the current rate of change of the Power of the Force, $\frac{dF}{dt}$?
(Plug in numbers but you don't need to simplify.)

$\frac{dF}{dt}=$ ____________________

12. $(14$ points$)$ Determine whether or not each of these limits exists. If it exists, find its value.

a. $\lim_{(x,y)\to(0,0)} \frac{3x^2y^2}{x^6+3y^3}$

○ Converges to _______ ○ Diverges Be sure to say why!

b. $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^2}$

○ Converges to _______ ○ Diverges Be sure to say why!

13. $(12$ points$)$ Find a scalar potential, f, for the vector field $\vec{F} = \langle \cos y, \sin z - x\sin y, 2z + y\cos z \rangle$.
(You MUST SHOW your derivation.)

$f =$ ___________________________________