Name_____

MATH 221 Exam 3 Spring 2023

Section 501 P. Yasskin

1	/15	4	/20
2	/15	5	/20
3	/15	6	/20
		Total	/105

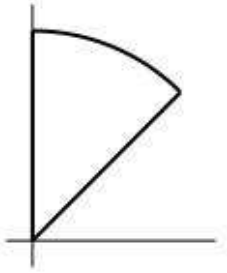
Work Out: (Points indicated. Part credit possible. Show all work.)

1. $(15$ points$)$ Given the vector field $\vec{F}(x,y,z) = \langle xz^2, yz^2, z^3 \rangle$, compute the triple integral $\iiint \vec{\nabla} \cdot \vec{F}\, dV$ of its divergence over the solid between $y = x^2$ and $y = 2x$ for $0 \le z \le 3$.

2. $(15$ points$)$ Given the function $f(x,y,z) = xy + 3z$ compute the vector line integral $\int_A^B \vec{\nabla} f \cdot d\vec{s}$ along the twisted cubic $\vec{r}(t) = \left(t, t^2, \frac{2}{3}t^3\right)$ between $A = \left(1, 1, \frac{2}{3}\right)$ and $B = (3, 9, 18)$.

3. (15 points) Compute $\int_0^{\sqrt{2}} \int_x^{\sqrt{4-x^2}} e^{-x^2-y^2}\, dy\, dx$

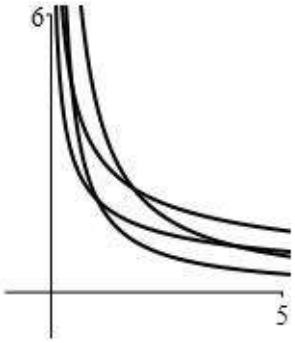
Hint: Change coordinates.



4. (20 points) Compute $\iint_D xy^2\, dA$ over the diamond shaped region in the first quadrant bounded by the curves

$$x = \frac{4}{y^2} \qquad x = \frac{9}{y^2} \qquad y = \frac{2}{x} \qquad y = \frac{4}{x}$$

HINT: Let $u = xy^2$ and $v = xy$. What are $\frac{y^2}{u}$ and $\frac{u}{v}$?



5. $(20$ points$)$ Consider the solid cylinder $x^2 + y^2 \leq 4$ for $2 \leq z \leq 6$ with density is $\delta = (x^2 + y^2)z$.

a. Find the mass of the cylinder.

b. Find the center of mass of the cylinder.

6. $(20$ points$)$ Given the vector field $\vec{F}(x,y,z) = \langle yz^2, -xz^2, z^3 \rangle$ compute the vector surface integral $\iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ along the side surface of the cylinder $x^2 + y^2 = 4$ for $2 \le z \le 6,$ oriented outward. (There are no ends on the cylinder.) Parametrize the cylinder by $\vec{R}(z,\theta) = (2\cos\theta, 2\sin\theta, z).$