Name_____ Section:_____

MATH 221 Exam 1, Version B Fall 2023

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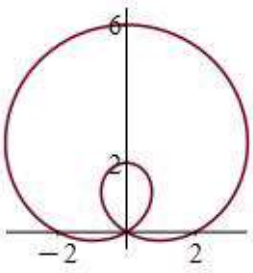
1-9	/54	12	/10
10	/20	13	/10
11	/10	Total	/104

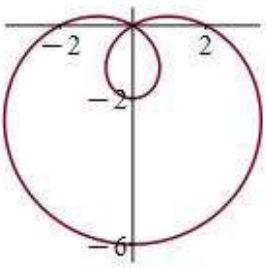
Multiple Choice: (6 points each. No part credit.)

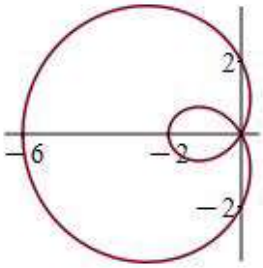
1. Find the sphere which is tangent to the z-axis whose center is $(4,3,2)$.

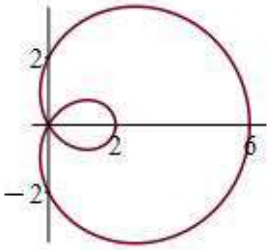
a. $(x-4)^2+(y-3)^2+(z-2)^2=4$
b. $(x-4)^2+(y+3)^2+(z-2)^2=25$
c. $(x-4)^2+(y-3)^2+(z-2)^2=25$
d. $(x+4)^2+(y+3)^2+(z+2)^2=4$
e. $(x+4)^2+(y+3)^2+(z+2)^2=25$

2. Which of the following is the plot of the polar curve $r=4\cos\theta-2$?

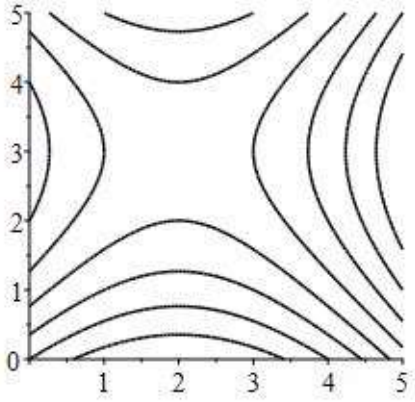
a. 

b. 

c. 

d. 

3. The plot at the right is the contour plot of which function?



a. $z=(x-2)^2+(y-3)^2$
b. $z=(x-2)^2-(y-3)^2$
c. $z=(x-3)^2+(y-2)^2$
d. $z=(x-3)^2-(y-2)^2$

4. The force $\vec{F} = \langle 7, -3 \rangle$ pushes a mass from $P = (12, 1)$ to $Q = (7, -1)$.
Find the angle between the force and the displacement.

a. $135°$
b. $120°$
c. $60°$
d. $45°$
e. $30°$

5. Do the vectors $\vec{u} = \langle 2, 0, 1 \rangle$, $\vec{v} = \langle 0, -1, 3 \rangle$ and $\vec{w} = \langle 3, 2, 0 \rangle$ form a left or right handed triplet?
Then find the volume of the parallelepiped with these edges.

a. left handed $V = 3$
b. left handed $V = 9$
c. left handed $V = -9$
d. right handed $V = 3$
e. right handed $V = -9$

6. Find an equation of the plane through the point $P = (3, 2, 1)$
which is perpendicular to the line $(x, y, z) = (1 + 4t, 2 + 3t, 3 + 2t)$.
Then find where the plane passes through the z-axis.

a. $z = 2$
b. $z = 4$
c. $z = 5$
d. $z = 10$
e. $z = 20$

7. Classify the quadratic curve: $x^2 - 6x = 2y^2 - 4y - 7$.

a. parabola opening in the x direction
b. parabola opening in the y direction
c. hyperbola opening up and down
d. hyperbola opening left and right
e. cross

8. Your drone flies NorthEast $5\sqrt{2}$ km and then East 7 km. If it flies home along a straight line, how far does it need to fly to get home?

a. 11 km
b. 12 km
c. $7 + 5\sqrt{2}$ km
d. 13 km
e. 17 km

9. Find the circulation in a bowl of water, counterclockwise around the circle $x^2 + y^2 = 16$, with $z = 3$, if its fluid velocity field is $\vec{V} = \langle x - y, x + y, 2z \rangle$.

a. 2π
b. 4π
c. 8π
d. 16π
e. 32π

Work Out: (Points indicated. Part credit possible. Show all work.)

10. (20 pts) Consider the twisted cubic $\vec{r} = (t^3, 3t^2, 6t)$. Compute each of the following.
Note: $t^4 + 4t^2 + 4 = (t^2 + 2)^2$

a. (6 pts) Arc length between $(0,0,0)$ and $(1,3,6)$.

b. (6 pts) Curvature $\kappa = \dfrac{|\vec{v} \times \vec{a}|}{|\vec{v}|^3}$.
HINT: Factor out an 18^2.

c. (4 pts) Tangential acceleration, a_T.
HINT: You do NOT need to compute $\hat{T}$, $\hat{N}$ or $\hat{B}$.

d. (4 pts) Normal acceleration, a_N.
HINT: You do NOT need to compute $\hat{T}$, $\hat{N}$ or $\hat{B}$.

11. (10 pts) Find the average value of the function $f(x,y,z) = x^2$ on the helix $\vec{r}(t) = (3\cos t, 3\sin t, 4t)$ for $0 \le t \le 2\pi$.

12. (10 pts) Write the vector $\vec{a} = \langle 5, 5 \rangle$ as the sum of vectors $\vec{p}$ and $\vec{q}$ where $\vec{p}$ is parallel to $\vec{b} = \langle 3, 1 \rangle$ and $\vec{q}$ is perpendicular to $\vec{b}$.

13. (10 pts) Consider the 2 planes:

$$P_1: \quad 2x + y + 3z = 8$$
$$P_2: \quad x + 2y - 2z = 7$$

Determine if they are parallel or intersecting. If they intersect, find a parametric equation for the line of intersection.

You MUST show why they are or are not parallel.