Name________________________________ Section:__________

MATH 221 Exam 2, Version C Fall 2023

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1-8	/48	10	/20
9	/36	Total	/104

Multiple Choice: (6 points each. No part credit.)

1. Consider the function $z = f(x,y) = xy^4$. Its y-trace at $x = 3$ is the intersection of the graph of $z = f(x,y)$ and the plane $x = 3$. Find the slope of this y-trace at $y = 2$.

a. 8

b. 24

c. 27

d. 54

e. 96

2. Consider the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^6 + y^2}$.

Which of the following paths of approach gives a different value of the limit?

a. $y = x^2$ & $x \to 0$

b. $y = x^3 + x^2$ & $x \to 0$

c. $y = x^3 + x^4$ & $x \to 0$

d. $y = x^4$ & $x \to 0$

e. They are all equal.

Hint: Don’t bother multiplying out any quadratic.

3. Find the plane tangent to the graph of the function $z = \dfrac{x^3}{y^2}$ at the point $(2,1)$.

Its z-intercept is

a. $c = -32$

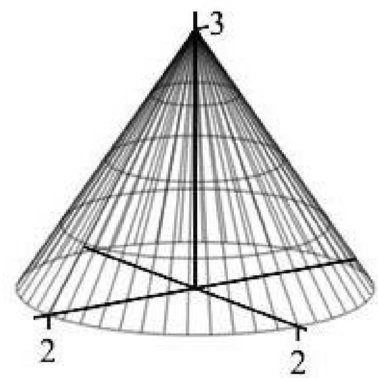
b. $c = -8$

c. $c = 0$

d. $c = 8$

e. $c = 32$

4. The volume of a cone is $V = \frac{1}{12}\pi D^2 H$ where D is the diameter and H is the height. Currently, $D = 4$ cm, $H = 3$ cm and $V = 4\pi$ cm. If V and H increase by $\Delta V = 0.6\pi$ cm and $\Delta H = 0.3$ cm, use the linear approximation to approximate how much D changes.



a. $\Delta D \approx 0.1$ cm

b. $\Delta D \approx 0.2$ cm

c. $\Delta D \approx 0.3$ cm

d. $\Delta D \approx 0.4$ cm

e. $\Delta D \approx 0.6$ cm

5. Let $f(x,y) = xe^{xy}$. Compute $f_{xy}(2, \ln 2)$.

a. $16 + 16\ln 2$

b. $4 + 4\ln 2$

c. $4 - 4\ln 2$

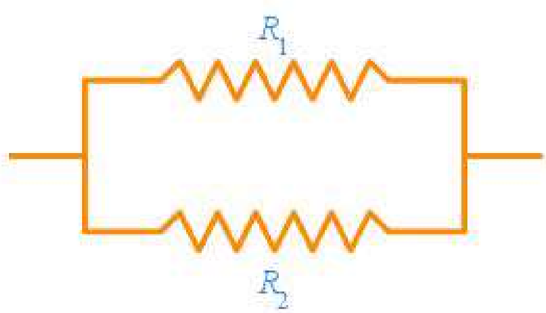
d. $2 + 2\ln 2$

e. $2 - 2\ln 2$

6. If two resistors, with resistances R_1 and R_2 are connected in parallel, then the total resistance R satisfies:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or} \quad R = \frac{R_1 R_2}{R_1 + R_2}$$

Currently $R_1 = 200$ ohms, $R_2 = 400$ ohms and $R = \frac{400}{3}$ ohms. If R_1 and R_2 are increasing at $\frac{dR_1}{dt} = 1.8\ \frac{\text{ohms}}{\text{min}}$ and $\frac{dR_2}{dt} = 0.9\ \frac{\text{ohms}}{\text{min}}$, at what rate is R changing?



a. $\frac{dR}{dt} = 0.1\ \frac{\text{ohms}}{\text{min}}$

b. $\frac{dR}{dt} = 0.3\ \frac{\text{ohms}}{\text{min}}$

c. $\frac{dR}{dt} = 0.6\ \frac{\text{ohms}}{\text{min}}$

d. $\frac{dR}{dt} = 0.9\ \frac{\text{ohms}}{\text{min}}$

e. $\frac{dR}{dt} = 1.8\ \frac{\text{ohms}}{\text{min}}$

7. Find the equation of the plane tangent to $x^2 z \sin y = 2\sqrt{2}$ at the point $P = \left(2, \frac{\pi}{4}, 1\right)$.
It's z-intercept is:

a. $c = 3 + \frac{\pi}{4}$

b. $c = 1 + \frac{\pi}{4}$

c. $c = 3 + \frac{\pi}{2}$

d. $c = 2 + \frac{\pi}{2}$

e. $c = 1 + \frac{\pi}{4}$

8. The point $(x,y) = (2,1)$ is a critical point of the function $f(x,y) = 12xy - x^3 - 8y^3$.
Use the Second Derivative Test to classify this critical point.

a. Local Minimum

b. Local Maximum

c. Inflection Point

d. Saddle Point

e. Test Fails

Work Out: (Points indicated. Part credit possible. Show all work.)

9. (36 pts) Obi-Two is flying the Centurion Eagle through the Force, Desperation and Luck fields. The Force, F, depends on the Desperation, D, and Luck, L, by the relation: $F = 3DL^2$. Currently, the Desperation and Luck and their gradients are:

$$D = 3 \qquad \vec{\nabla} D = \langle 3,0,1 \rangle$$

$$L = 2 \qquad \vec{\nabla} L = \langle 1,2,0 \rangle$$

a. (23 pts) Obi-Two's current velocity is $\vec{v} = \langle 1,2,3 \rangle$. Find the rate that Obi-Two sees the Force changing.

b. (13 pts) In what direction should Obi-Two travel to increase the Force as fast as possible? HINT: Compute each (x,y,z) partial derivative separately.

10. (20 pts) Find the point on the surface $z = \dfrac{1}{4x^2y^4}$ in the 1^{st} octant which is closest to the origin.

HINT: Write the constraint as $g = x^2y^4z = \dfrac{1}{4}$.