

Name _____ Section: _____

MATH 221 Exam 3, Version B

Fall 2023

502,503

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Multiple Choice: (6 points each. No part credit.)

| | | | |
|-----|-----|-------|------|
| 1-8 | /48 | 10 | /16 |
| 9 | /16 | 11 | /24 |
| | | Total | /104 |

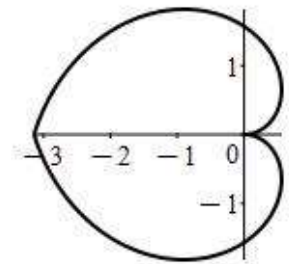
- Find the divergence of the vector field $\vec{F} = \langle x^2y, y^2z, z^2x \rangle$ and evaluate the divergence at $P = (1, 2, 3)$.
 - 2
 - 20
 - 22
 - 23
 - 24

- Find the curl of the vector field $\vec{F} = \langle x^2y, y^2z, z^2x \rangle$ and evaluate the curl at $P = (1, 2, 3)$.
 - $\langle -4, -9, -1 \rangle$
 - $\langle -4, 9, -1 \rangle$
 - $\langle 2, -12, 9 \rangle$
 - $\langle 2, 12, 9 \rangle$
 - $\langle 1, -4, 9 \rangle$

- Let f be a scalar potential for $\vec{F} = \langle yz + 2x, xz + 2y, xy + 2z \rangle$. Compute $f(1, 2, 3) - f(0, 0, 0)$. (Note: The subtraction cancels off the arbitrary constant.)
 - 2
 - 20
 - 22
 - 23
 - 24

4. Use a Riemann sum with 6 squares evaluated at the center of each square to estimate the volume of the solid over the rectangle $[1, 7] \times [2, 6]$ below the surface $f = x^2 + y^2$.
- 130
 - 214
 - 520
 - 856
 - 872

5. Find the area inside the heart which in polar coordinates is the spiral $r = |\theta|$ for $-\pi \leq \theta \leq \pi$.
HINT Double the area inside half the spiral.



- $\frac{\pi^3}{6}$
- $\frac{\pi^3}{3}$
- $\frac{\pi^3}{2}$
- $\frac{\pi^4}{4}$
- $\frac{\pi^4}{2}$

6. Find mass of the solid below $z = 25 - x^2 - y^2$
 above the xy -plane inside the cylinder $x^2 + y^2 = 9$
 if the volume density is $\delta = z$.



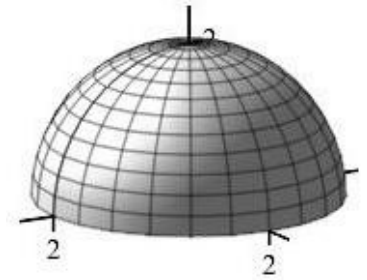
- a. $\frac{\pi}{6}(25^3 - 16^3)$
 b. $\frac{\pi}{6}(25^3 + 16^3)$
 c. $\pi\left(25^2 \cdot 3 - 50 \cdot 3^2 + \frac{3^5}{5}\right)$
 d. $\pi\left(25^2 \cdot 3 + 50 \cdot 3^2 + \frac{3^5}{5}\right)$
 e. $\pi\left(25\frac{3^2}{2} - \frac{3^4}{4}\right)$

7. Find the Jacobian factor for the 3D coordinate system:

$$(x, y, z) = \vec{R}(u, v, w) = (vw, uw, uv) \quad \text{with } u > 0, \quad v > 0, \quad w > 0$$

- a. uvw
 b. $2uvw$
 c. $u + v + w$
 d. $2u + 2v + 2w$
 e. $vw + uw + uv$

8. Find the average value of the function $f = x^2 + y^2 + z^2$ on the solid hemisphere $0 \leq z \leq \sqrt{4 - x^2 - y^2}$.



- a. $\frac{5}{5}$
- b. $\frac{4}{5}\pi$
- c. $\frac{12}{5}$
- d. $\frac{16}{3}\pi$
- e. $\frac{64}{5}\pi$

Work Out: (Points indicated. Part credit possible. Show all work.)

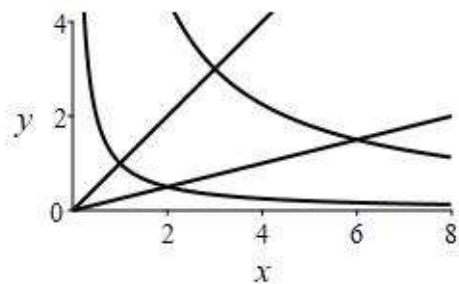
9. (16 points) Consider a plate bounded by $y = 4 - x^2$ and the x -axis with surface density $\delta = x^2$.
- a. (8 pts) Find the mass of a plate.

 - b. (8 pts) Find the center of mass of a plate.

10. (16 points) Compute $\iint_D x^2 dA$ over the diamond shaped region in the 1st quadrant bounded by
- $$y = \frac{1}{x} \quad y = \frac{9}{x} \quad y = x \quad y = \frac{1}{4}x$$

HINT: Use the curvilinear coordinates

$$x = uv \quad y = \frac{v}{u}.$$



- a. (5 pts) Find the Jacobian factor.

- b. (1 pts) Express the integrand in terms of the coordinates.

- c. (4 pts) Substitute $x = uv$ and $y = \frac{v}{u}$ into the boundaries to express them in terms of u and v .

- d. (6 pts) Compute the integral.

11. (24 points) Consider the cone surface $z = 2\sqrt{x^2 + y^2}$ for $z \leq 6$ which may be parametrized by

$$\vec{R}(r, \theta) = \langle r \cos \theta, r \sin \theta, 2r \rangle$$

- a. (6 pts) Find the tangent vectors:

$$\vec{e}_r =$$

$$\vec{e}_\theta =$$

- b. (3 pts) Find the normal vector oriented down and out:

$$\vec{N} =$$

- c. (2 pts) Find the length of the normal vector:

$$|\vec{N}| =$$

- d. (4 pts) Find the mass of the cone if the mass density is $\delta = \sqrt{x^2 + y^2}$.

$$M =$$

- e. (3 pts) Find the **curl** of the vector field $\vec{F} = \langle yz, -xz, z^2 \rangle$ in rectangular coordinates:

$$\vec{\nabla} \times \vec{F} =$$

- f. (2 pts) Evaluate the **curl** of \vec{F} on the cone:

$$\vec{\nabla} \times \vec{F} \Big|_{\vec{R}} =$$

- g. (4 pts) Find the flux of the **curl** of \vec{F} down and out of the cone:

$$\iint_C (\vec{\nabla} \times \vec{F}) \cdot d\vec{S} =$$