Name_____________________________ Section:__________

MATH 221 Final Exam, Version A Fall 2023

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1-12	/60	14	/10
13	/10	15	/24
		Total	/104

Multiple Choice: (5 points each. No part credit.)

1. Consider the triangle with vertices $A = (2,4)$, $B = (1,1)$ and $C = (0,3)$.
Find the angle at B.

a. $30°$

b. $45°$

c. $60°$

d. $120°$

e. $135°$

2. Find the arc length of the curve $\vec{r}(t) = \langle e^t, 2t, 2e^{-t} \rangle$ between $(1,0,2)$ and $(e,2,2e^{-1})$.
Hint: Look for a perfect square.

a. $e + 2e^{-1}$

b. $e + 2e^{-1} - 3$

c. $e - 2e^{-1}$

d. $e - 2e^{-1} + 1$

e. $e - 2e^{-1} - 1$

3. Find the point where the lines $(x,y,z) = (3-t, 2+t, 2t)$ and $(x,y,z) = (-1+2t, 5-t, 3+t)$ intersect. At this point $x+y+z =$

a. $\frac{17}{2}$

b. 9

c. $\frac{25}{3}$

d. 11

e. They do not intersect.

4. Find the equation of the plane tangent to the graph of $f(x,y) = x^2y + xy^2$ at the point $(2,1)$. Its z-intercept is

a. -12

b. -6

c. 0

d. 6

e. 12

5. Find the plane tangent to the surface $x^2z^2 + y^4 = 5$ at the point $(2,1,1)$.

a. $2x + y + z = 6$

b. $2x + y + z = 5$

c. $x + y + 2z = 5$

d. $x - y + 2z = 3$

e. $x - y + 2z = 6$

6. The dimensions of a closed rectangular box are measured as 70 cm, 50 cm and 40 cm with a possible error of 0.2 cm in each dimension. Use differentials to estimate the maximum error in the calculated surface area of the box.

a. 8

b. 16

c. 32

d. 64

e. 128

7. Compute the line integral $\int -y\,dx + x\,dy$ **clockwise** around the semicircle $x^2 + y^2 = 9$ from $(-3,0)$ to $(3,0)$.
HINT: Parametrize the curve.

a. -9π

b. -3π

c. π

d. 3π

e. 9π

8. Compute $\int \vec{F} \cdot d\vec{s}$ for $\vec{F} = (y,x)$ along the curve $\vec{r}(t) = \left(e^{\cos(t^2)}, e^{\sin(t^2)}\right)$ for $0 \le t \le \sqrt{\pi}$.
HINT: Find a scalar potential.

a. $e - \frac{1}{e}$

b. $\frac{1}{e} - e$

c. $\frac{2}{e}$

d. $2e$

e. 0

9. Compute $\oint_S \vec{F} \cdot d\vec{s}$ for $\vec{F} = (-xy^2 + x^3, x^2y - y^3)$
counterclockwise around the square $[0,3] \times [0,2]$.
Hint: Use a theorem.

a. 6

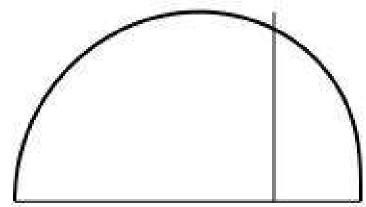
b. 12

c. 16

d. 24

e. 36

10. Find the mass of the region inside the upper half of the limaçon $r = 2 - \cos\theta$ if the surface density is $\delta = y$.



a. $\frac{20}{3}$

b. $\frac{15}{3}$

c. $\frac{13}{3}$

d. $\frac{10}{3}$

e. $\frac{5}{3}$

11. Consider the vector field $\vec{F} = \vec{\nabla} \times \vec{G}$ where $\vec{G} = \langle x^2 z, y^2 z, x^3 + y^3 \rangle$.
In which quadrant is $\vec{F}$ **always diverging**?

a. I

b. II

c. III

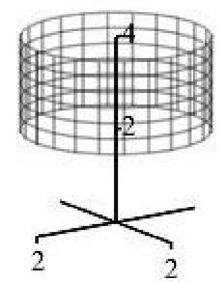
d. IV

e. None of them

12. Consider the cylinder C given by $x^2 + y^2 = 4$ for $2 \le z \le 4$ with normal pointing outward.
Let T be the top circle and B be the bottom circle both oriented counterclockwise as seen from above.
For a certain vector field $\vec{F}$ we have:

$$\iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S} = 14 \quad \text{and} \quad \oint_B \vec{F} \cdot d\vec{s} = 3$$

Compute $\oint_T \vec{F} \cdot d\vec{s}$.



a. 17

b. 11

c. 8

d. -11

e. -17

Work Out: (Points shown. Part credit possible. Show all work.)

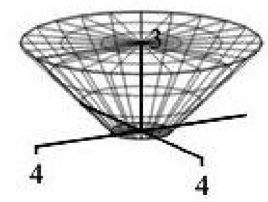
13. (10 points) Find the dimensions and volume of the largest box which sits on the xy-plane and whose upper vertices are on the elliptic paraboloid $z + 2x^2 + 3y^2 = 12$.

You do not need to show it is a maximum.

14. (10 points) Find the mass and center of mass of the conical **surface** $z = \sqrt{x^2 + y^2}$ for $z \le 2$ with density $\delta = x^2 + y^2$. The cone may be parametrize as $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r)$.

15. (24 points) Verify Gauss' Theorem

$$\iiint_V \vec{\nabla} \cdot \vec{F}\, dV = \iint_{\partial V} \vec{F} \cdot d\vec{S}$$



for the vector field $\vec{F} = \langle x, y, 2z \rangle$ and the solid bowl filled with water bounded by the surfaces

$z = 0$ and $z = 3$ and $r = z + 1$.

Be sure to check orientations. Use the following steps:

Left Hand Side:

a. Compute the divergence of $\vec{F}$:

b. Compute the left hand side: (Be careful with the bounds on r and z.)

Right Hand Side: The boundary surface consists of two disks and the side surface.

Side Surface S:

The sides may be parametrized by $\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r-1)$

c. Compute the normal vector and check its orientation:

d. Evaluate $\vec{F} = \langle x, y, 2z \rangle$ on the sides:

e. Compute their dot product:

f. Compute the flux through S: (Be careful with the bounds on r.)

(continued)

Top Disk T:

g. Parametrize the top disk T. (Start from cylindrical coordinates.)

h. Compute the normal vector and check its orientation:

i. Evaluate $\vec{F} = \langle x, y, 2z \rangle$ on the top disk:

j. Compute the flux through T: (Be careful with the bounds on r.)

Bottom Disk B:

k. Parametrize the bottom disk B. (Start from cylindrical coordinates.)

l. Compute the normal vector and check its orientation:

m. Evaluate $\vec{F} = \langle x, y, 2z \rangle$ on the bottom disk:

n. Compute the flux through B: (Be careful with the bounds on r.)

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o. Compute the **TOTAL** right hand side: