Name________________________ Section:__________

MATH 221 Final Exam, Version B Fall 2023

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|---|---|---|---|
| 1-12 | /60 | 14 | /10 |
| 13 | /10 | 15 | /24 |
| | | Total | /104 |

Multiple Choice: (5 points each. No part credit.)

**1**. Which of the following lines lies in the plane: $2x-y-z=0$ ?

**a**. $(x,y,z)=(1,2,3)+t(1,1,1)$

**b**. $(x,y,z)=(3,2,1)+t(1,1,1)$

**c**. $(x,y,z)=(2,1,3)+t(1,1,1)$

**d**. $(x,y,z)=(3,1,2)+t(1,1,1)$

**e**. $(x,y,z)=(1,3,2)+t(1,1,1)$

**2**. Find the arc length of the curve $\vec{r}(t)=(2t^2,t^3)$ between $(0,0)$ and $(2,1)$.

**a**. $\frac{31}{27}$

**b**. $\frac{61}{27}$

**c**. $\frac{91}{27}$

**d**. $\frac{31}{9}$

**e**. $\frac{61}{9}$

**3**. Find the point where the line $\frac{x-4}{3} = \frac{y-4}{2} = z-4$ intersects the plane $3x+2y+z = 10$.
At this point $x+y+z =$

**a**. $0$
**b**. $2$
**c**. $4$
**d**. $6$
**e**. They do not intersect.

**4**. Find the equation of the plane tangent to the graph of $z = 3x^2y - 2y^3$ at the point $(2,1)$.
Its $z$-intercept is

**a**. $-20$
**b**. $-14$
**c**. $14$
**d**. $20$
**e**. $40$

**5**. Find the equation of the line perpendicular to the graph of $x^3y^2z - 2x^2z^2 = 10$ at the point $(1,3,2)$.
This line intersects the $xy$-plane at:

**a**. $\left(-\frac{19}{3},2,0\right)$
**b**. $\left(2,\frac{19}{3},0\right)$
**c**. $(-75,-21,0)$
**d**. $\left(\frac{19}{3},-2,0\right)$
**e**. $(21,-75,0)$

**6**. The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

The radius $r$ is currently $3$ cm and is increasing at $2$ cm/sec.

The height $h$ is currently $4$ cm and is decreasing at $1$ cm/sec.

Is the volume increasing or decreasing and at what rate?

**a**. decreasing at $19\pi$ cm$^3/\text{sec}$

**b**. decreasing at $13\pi$ cm$^3/\text{sec}$

**c**. neither increasing nor decreasing

**d**. increasing at $19\pi$ cm$^3/\text{sec}$

**e**. increasing at $13\pi$ cm$^3/\text{sec}$

**7**. Compute the line integral $\int -y\,dx + x\,dy$ along the parabola $y = x^2$ from $(1,1)$ to $(2,4)$.

HINT: Parametrize the curve.

**a**. $\frac{7}{3}$

**b**. $\frac{5}{3}$

**c**. $\frac{1}{3}$

**d**. $1$

**e**. $3$

**8**. Compute $\int_{\vec{r}} \vec{F} \cdot d\vec{s}$ for $\vec{F} = (2x + y + z, 2y + x + z, 2z + x + y)$ along the curve $\vec{r}(t) = (t\cos t, t\sin t, t e^{t/\pi})$ between $t = 0$ and $t = \pi$.

HINT: Find a scalar potential.

**a**. $\pi^2(1 + e^2 - e)$

**b**. $\pi^2(1 + e^2 - 2e)$

**c**. $\pi^2(1 + e^2 + e)$

**d**. $\pi^2(1 + e^2 + 2e)$

**e**. $\pi^2(1 + e^2)$

9. Compute $\oint_C \vec{F} \cdot d\vec{s}$ for $\vec{F} = (-x^2y + x^3 - y^3, xy^2 + x^3 - y^3)$
counterclockwise around the circle $x^2 + y^2 = 9$.
HINT: Use a theorem.

a. $324\pi$

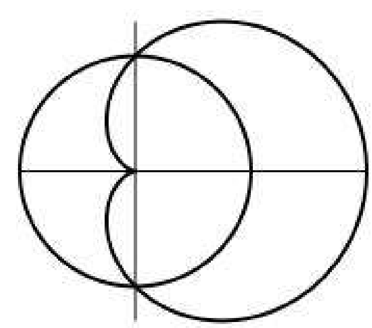
b. $162\pi$

c. $144\pi$

d. $72\pi$

e. $36\pi$

10. Find the area inside the cardioid $r = 1 + \cos\theta$
but outside the circle $r = 1$.



a. $\frac{\pi}{4}$

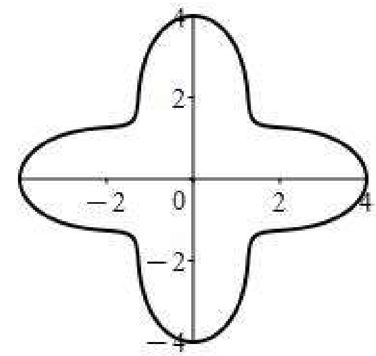
b. $\frac{\pi}{2}$

c. $2 - \frac{\pi}{4}$

d. $2 + \frac{\pi}{4}$

e. $2 - \frac{\pi}{2}$

**11**. Compute $\oint \vec{\nabla} f \cdot d\vec{s}$ counterclockwise once around the polar curve $r = 3 + \cos(4\theta)$ for the function $f(x,y) = x^2 y$.




**a**. $2\pi$

**b**. $4\pi$

**c**. $6\pi$

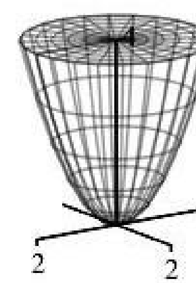
**d**. $8\pi$

**e**. $0$

**12**. Consider the parabolic surface $P$ given by $z = x^2 + y^2$ for $z \le 4$ with normal pointing up and in, the disk $D$ given by $x^2 + y^2 \le 4$ and $z = 4$ with normal pointing up, and the volume $V$ between them. For a certain vector field $\vec{F}$ we have:

$$\iiint_V \vec{\nabla} \cdot \vec{F}\, dV = 14 \quad \text{and} \quad \iint_D \vec{F} \cdot d\vec{S} = 3$$

Compute $\iint_P \vec{F} \cdot d\vec{S}$.




**a**. $17$

**b**. $11$

**c**. $8$

**d**. $-11$

**e**. $-17$

Work Out: (Points shown. Part credit possible. Show all work.)

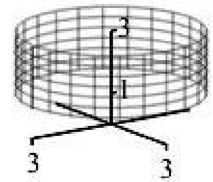
**13**. (10 points) Find $3$ numbers $a$, $b$ and $c$ whose sum is $12$ for which $ab + 2ac + 3bc$ is a maximum.

You do not need to show it is a maximum.

**14**. (10 points) Find the mass and center of mass of the cylindrical **surface** $x^2 + y^2 = 9$ for $0 \leq z \leq 2$ with density $\delta = z$. The cylinder may be parametrize as $\vec{R}(\theta, z) = (3\cos\theta, 3\sin\theta, z)$.

**15**. (24 points) Verify Stokes' Theorem

$$\iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint_{\partial C} \vec{F} \cdot d\vec{s}$$




for the vector field $\vec{F} = (yz^2, -xz^2, z^3)$ and the cylinder $x^2 + y^2 = 9$ for $1 \le z \le 2$ oriented outward.
Be sure to check orientations. Use the following steps:

**Left Hand Side**: The cylindrical surface may be parametrized by $\vec{R}(\theta, z) = (3\cos\theta, 3\sin\theta, z)$.

**a**. Compute the normal vector and check its orientation:

**b**. Compute the curl of $\vec{F}$ and evaluate it on the cylinder.

**c**. Compute the dot product of the curl of $\vec{F}$ and the normal:

**d**. Compute the surface integral:

(continued)

**Right Hand Side**: Let $U$ be the upper circle and $L$ be the lower circle.

**e**. Parametrize $U$. Find the velocity and check its orientation:

**f**. Evaluate $\vec{F} = (yz^2, -xz^2, z^3)$ on the circle and compute its dot product with the velocity:

**g**. Compute the line integral $\oint_U \vec{F} \cdot d\vec{s}$

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**h**. Parametrize $L$. Find the velocity and check its orientation:

**i**. Evaluate $\vec{F} = (yz^2, -xz^2, z^3)$ on the circle and compute its dot product with the velocity:

**j**. Compute the line integral $\oint_L \vec{F} \cdot d\vec{s}$

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**k**. Combine $\oint_U \vec{F} \cdot d\vec{s}$ and $\oint_L \vec{F} \cdot d\vec{s}$ to get $\oint_{\partial C} \vec{F} \cdot d\vec{s}$.