

Name \_\_\_\_\_

MATH 221      Final Exam Version H      Fall 2019

Section 204      P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1-11	/55	13	/10
12	/15	14	/25
		Total	/105

1. A triangle has vertices  $A = (3,2,1)$ ,  $B = (3,3,2)$  and  $C = (4,4,2)$ . Find the angle at  $A$ .

- a.  $0^\circ$
- b.  $30^\circ$
- c.  $45^\circ$
- d.  $60^\circ$
- e.  $90^\circ$

2. Find the area of the triangle with vertices  $A = (3,2,1)$ ,  $B = (3,3,2)$  and  $C = (4,4,2)$ .

- a.  $2\sqrt{6}$
- b.  $\sqrt{6}$
- c.  $\frac{\sqrt{6}}{2}$
- d.  $\sqrt{3}$
- e.  $\frac{\sqrt{3}}{2}$

3. Find the arc length of the curve  $\vec{r}(t) = (t^2, 2t, \ln t)$  between  $(1, 2, 0)$  and  $(e^2, 2e, 1)$ .

- a.  $e^2 + 1$
- b.  $e^2 - 1$
- c.  $e^2$
- d.  $\sqrt{e^2 + 1}$
- e.  $\sqrt{e^2 + 1} - 1$

4. Find the average value of the function  $f(x, y, z) = x$  along the curve  $\vec{r}(t) = (t^2, 2t, \ln t)$  between  $(1, 2, 0)$  and  $(e^2, 2e, 1)$ .

- a.  $\frac{e^2}{2} - \frac{1}{e^2}$
- b.  $\frac{e^4}{2} - \frac{e^2}{2} - 1$
- c.  $\frac{e^2}{2} - \frac{1}{2} - \frac{1}{e^2}$
- d.  $\frac{e^4}{2} + \frac{e^2}{2} - 1$
- e.  $\frac{e^2}{2} + \frac{1}{2} - \frac{1}{e^2}$

5. Find the plane tangent to  $x \sin y + 2z \cos y = 2$  at  $(x, y, z) = \left(\sqrt{2}, \frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$ . The  $z$ -intercept is:

- a. 2
- b.  $\sqrt{2}$
- c.  $\frac{1}{\sqrt{2}}$
- d.  $\frac{1}{2\sqrt{2}}$
- e. 0

6. The velocity field of the water in a sink is  $\vec{V} = \langle -x^2y, xy^2 \rangle$ . Find the circulation of the water,  $\text{Circ} = \oint \vec{V} \cdot d\vec{s}$ , counterclockwise around the circle  $x^2 + y^2 = 4$ .
- $2\pi$
  - $\frac{16}{3}\pi$
  - $8\pi$
  - $32\pi$
  - $64\pi$

7. Compute  $\int \vec{F} \cdot d\vec{s}$  for  $\vec{F} = \langle 4x^3, 3y^2, 2z \rangle$  along the curve  $\vec{r}(t) = \left( t \sin\left(\frac{\pi}{2}t\right), 2^t \cos\left(\frac{\pi}{2}t\right), 2^t \sin\left(\frac{\pi}{2}t\right) \right)$  from  $t = 0$  to  $t = 1$ .
- 0
  - 1
  - 2
  - 3
  - 4

8. Compute  $\iint_S \vec{F} \cdot d\vec{S}$  for  $\vec{F} = \langle 0, 0, -2 \rangle$  over the surface  $\vec{R}(r, \theta) = (r \cos \theta, r \sin \theta, 16 - r^4)$  for  $0 \leq r \leq 2$  and  $0 \leq \theta \leq 2\pi$  oriented up and out.  
HINT: Find a vector potential.
- $-8\pi$
  - $-4\pi$
  - $-2\pi$
  - $2\pi$
  - $4\pi$

9. Find the centroid of the solid hemisphere  $0 \leq z \leq \sqrt{4 - x^2 - y^2}$ .

- a.  $(0, 0, \frac{1}{2})$
- b.  $(0, 0, \frac{3}{4})$
- c.  $(0, 0, \frac{4}{3})$
- d.  $(0, 0, 4\pi)$
- e.  $(0, 0, 2\pi)$

10. Find the area of the surface  $\vec{R}(u, v) = (u, v, u + v)$  for  $0 \leq u \leq 2$  and  $0 \leq v \leq 3$ .

- a.  $12\sqrt{2}$
- b.  $12\sqrt{3}$
- c.  $6\sqrt{2}$
- d.  $6\sqrt{3}$
- e.  $3\sqrt{2}$

11. Find the flux of  $\vec{F} = \langle y, -z, x \rangle$  upward thru the surface  $\vec{R}(u, v) = (u, v, u + v)$  for  $0 \leq u \leq 2$  and  $0 \leq v \leq 3$ .

- a. 30
- b. 24
- c. 18
- d. 12
- e. 0

Work Out: (Points indicated. Part credit possible. Show all work.)

12. (15 points) The Ideal Gas Law says the Pressure,  $P$ , Density,  $\delta$ , and Temperature,  $T$ , are related by  $P = k\delta T$  for some constant  $k$ . We will assume the atmosphere is an ideal gas with  $k = 2$ . A weather balloon measures the Density and Temperature to be

$$\delta = 0.1 \frac{\text{gm}}{\text{m}^3} \quad T = 270^\circ\text{K}$$

and their gradients to be

$$\vec{\nabla}\delta = \langle 0.01, 0.02, 0.03 \rangle \quad \vec{\nabla}T = \langle 3, 1, -1 \rangle$$

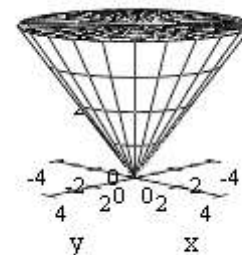
Find the gradient of the Pressure.

13. (10 points) Find the largest value of  $f = xy^2z^3$  on the plane  $2x + 3y + 6z = 72$ .

14. (25 points) Verify Gauss' Theorem  $\iiint_V \vec{\nabla} \cdot \vec{F} \, dV = \iint_{\partial V} \vec{F} \cdot d\vec{S}$

for the vector field  $\vec{F} = (xz, yz, 3z\sqrt{x^2 + y^2})$  and the solid cone  $\sqrt{x^2 + y^2} \leq z \leq 4$ .

Be careful with orientations. Use the following steps:



**First the Left Hand Side:**

a. Compute the divergence:

$$\vec{\nabla} \cdot \vec{F} =$$

b. Express the divergence and the volume element in the appropriate coordinate system:

$$\vec{\nabla} \cdot \vec{F} = \qquad dV =$$

c. Compute the left hand side:

$$\iiint_V \vec{\nabla} \cdot \vec{F} \, dV =$$

**Second the Right Hand Side:**

The boundary surface consists of a hemisphere  $H$  and a disk  $D$  with appropriate orientations.

d. Parametrize the disk  $D$ :

$$\vec{R}(r, \theta) =$$

e. Compute the tangent vectors:

$$\vec{e}_r =$$

$$\vec{e}_\theta =$$

f. Compute the normal vector:

$$\vec{N} =$$

g. Evaluate  $\vec{F} = (xz, yz, 3z\sqrt{x^2 + y^2})$  on the disk:

$$\vec{F}|_{\vec{R}(r,\theta)}$$

h. Compute the dot product:

$$\vec{F} \cdot \vec{N} =$$

i. Compute the flux through  $D$ :

$$\iint_D \vec{F} \cdot d\vec{S} =$$

j. Parametrize the cone  $C$ :

$$\vec{R}(r,\theta) =$$

k. Compute the tangent vectors:

$$\vec{e}_r =$$

$$\vec{e}_\theta =$$

l. Compute the normal vector:

$$\vec{N} =$$

m. Evaluate  $\vec{F} = (xz, yz, 3z\sqrt{x^2 + y^2})$  on the cone:

$$\vec{F}|_{\vec{R}(r,\theta)} =$$

n. Compute the dot product:

$$\vec{F} \cdot \vec{N} =$$

o. Compute the flux through  $C$ :

$$\iint_C \vec{F} \cdot d\vec{S} =$$

p. Compute the **TOTAL** right hand side: