Name_____ ID_____

MATH 251   Exam 1   Fall 2005

Sections 503   P. Yasskin

Multiple Choice: (6 points each. No part credit.)

1. Find the angle between the vectors $\vec{u} = \langle 1, -1, 1 \rangle$ and $\vec{v} = \langle 1, 2, 1 \rangle$.

   a. $0°$

   b. $30°$

   c. $45°$

   d. $60°$

   e. $90°$

2. If $p(x,y) = e^{x/y}$, find $\dfrac{\partial^2 p}{\partial y^2}$.

   a. $\dfrac{x}{y^2} e^{x/y}$

   b. $\dfrac{-1}{y^2} e^{x/y}$

   c. $\left(\dfrac{2x}{y^3} + \dfrac{x^2}{y^4} \right) e^{x/y}$

   d. $\left(\dfrac{2}{y^3} + \dfrac{1}{y^4} \right) e^{\frac{x}{y}}$

   e. $\left(\dfrac{2}{y^3} - \dfrac{1}{y^4} \right) e^{\frac{x}{y}}$

3. Find the line through the point $P = (2,-1,4)$ in the direction $\vec{v} = (1,3,-2)$. Where does this line intersect the xy-plane?

    a. $(2,-1,0)$

    b. $(4,5,0)$

    c. $(3,2,2)$

    d. $(3,2,0)$

    e. $(2,2.5,0)$

4. At $t = 4$ the velocity of a fly is $\vec{v} = (0,2,1)$, and its acceleration is $\vec{a} = (1,0,1)$. Find the unit binormal vector $\hat{B}$ to it's path.

    a. $\left(\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}\right)$

    b. $(2,-1,-2)$

    c. $\left(\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}\right)$

    d. $(2,1,-2)$

    e. $\left(\frac{2}{3}, \frac{1}{3}, \frac{-2}{3}\right)$

5. Find the equation of the plane tangent to the graph of $z = x\cos y - \sin(xy)$ at the point where $(x,y) = (1,\pi)$. Where does this plane intersect the z-axis?

    a. -1

    b. 0

    c. 1

    d. π

    e. -2π

6. Find the equation of the plane tangent to the surface $x^2z + y^2z^2 = 5$ at the point $(2,-1,1)$.

a. $2x - y + 3z = 8$

b. $2x - y + 3z = -8$

c. $2x + y + 3z = 6$

d. $2x + y + 3z = -6$

e. $4x - 2y + 6z = -8$

7. Duke Skywater is travelling through the galaxy. At the point with galactic coordinates $(40,25,53)$ (in lightyears), he measures the polaron density to be $U = 4300$ polarons/cm^3 and its gradient to be $\vec{\nabla}U = (3,2,1)$ polarons/cm^3/lightyear. Use this information to estimate the polaron density at the point with galactic coordinates $(42,26,52)$.

a. 4291

b. 4293

c. 4307

d. 4309

e. 4311

Work Out: (20 points each. Part credit possible. Show all work.)

8. $\left(20 \text{ points}\right)$ The temperature in a frying pan is given by $T(x,y) = 30 - \frac{x^2}{4} - \frac{y^4}{100}$ where distance is in cm and temperature is in $°C$. An ant is currently located at the point $(2,5)$ cm and has velocity $\vec{v} = (0.3, 0.1)$ cm/sec.

    a. What is the time rate of change of the temperature as seen by the ant?

    b. In what direction should the ant walk to decrease the temperature as fast as possible.

9. $\left(20 \text{ points}\right)$ Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex on the plane $x + \frac{y}{2} + \frac{z}{4} = 1.$
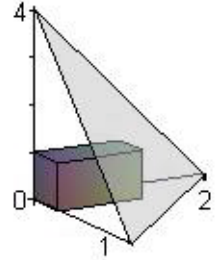
10. (20 points) Determine whether each of the following limits exists and say why or why not. If the limit exists, find it.

a. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^2}$

b. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$