Name_____ ID_____

MATH 251 Quiz 2 Fall 2005

Sections 503 Solutions P. Yasskin

Total	/34

All Work Out: (2 points each, includes 9 points extra credit)

Consider the parametric curve with position vector $\vec{r} = \left(t, t^2, \frac{2}{3}t^3\right)$.

Compute each of the following:

1. velocity

$\vec{v} = (1, 2t. 2t^2)$

2. acceleration

$\vec{a} = (0, 2, 4t)$

3. jerk

$\vec{j} = (0, 0, 4)$

4. length of velocity Simplify. (Note the quantity in the square root is a perfect square.)

$$|\vec{v}| = \sqrt{1 + 4t^2 + 4t^4} = \sqrt{(1 + 2t^2)^2} = 1 + 2t^2$$

5. speed

$$\frac{ds}{dt} = |\vec{v}(t)| = 1 + 2t^2$$

6. arclength between the points $(0, 0, 0)$ and $\left(1, 1, \frac{2}{3}\right)$

$$L = \int_{(0,0,0)}^{(1,1,2/3)} ds = \int_0^1 |\vec{v}|\, dt = \int_0^1 (1 + 2t^2)\, dt = \left[t + \frac{2}{3}t^3\right]_0^1 = \frac{5}{3}$$

7. unit tangent vector

$$\hat{T} = \frac{\vec{v}}{|\vec{v}|} = \frac{1}{1 + 2t^2}(1, 2t, 2t^2) = \left(\frac{1}{1 + 2t^2}, \frac{2t}{1 + 2t^2}, \frac{2t^2}{1 + 2t^2}\right)$$

8. cross product of velocity and acceleration

$$\vec{v} \times \vec{a} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2t & 2t^2 \\ 0 & 2 & 4t \end{vmatrix} = \hat{\imath}(8t^2 - 4t^2) - \hat{\jmath}(4t) + \hat{k}(2) = (4t^2, -4t, 2)$$

9. length of cross product of velocity and acceleration

$$|\vec{v} \times \vec{a}| = \sqrt{16t^4 + 16t^2 + 4} = 2\sqrt{4t^4 + 4t^2 + 1} = 2(1 + 2t^2)$$

10. unit binormal

$$\hat{B} = \frac{\vec{v}\times\vec{a}}{|\vec{v}\times\vec{a}|} = \frac{1}{2(1+2t^2)}(4t^2,-4t,2) = \frac{1}{1+2t^2}(2t^2,-2t,1) = \left(\frac{2t^2}{1+2t^2},\frac{-2t}{1+2t^2},\frac{1}{1+2t^2}\right)$$

11. unit principal normal

$$\hat{N} = \hat{B}\times\hat{T} = \frac{1}{(1+2t^2)^2}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2t^2 & -2t & 1 \\ 1 & 2t & 2t^2 \end{vmatrix} = \frac{1}{(1+2t^2)^2}\left[\hat{\imath}(-4t^3-2t) - \hat{\jmath}(4t^4-1) + \hat{k}(4t^3+2t)\right]$$

$$= \frac{1}{(1+2t^2)^2}(-4t^3-2t, 1-4t^4, 4t^3+2t)$$

Optional: $\hat{N} = \frac{1}{(1+2t^2)^2}(-2t(2t^2+1),(1-2t^2)(1+2t^2),2t(2t^2+1))$

$$= \frac{1}{1+2t^2}(-2t,1-2t^2,2t) = \left(\frac{-2t}{1+2t^2},\frac{1-2t^2}{1+2t^2},\frac{2t}{1+2t^2}\right)$$

12. curvature

$$\kappa = \frac{|\vec{v}\times\vec{a}|}{|\vec{v}|^3} = \frac{2(1+2t^2)}{(1+2t^2)^3} = \frac{2}{(1+2t^2)^2}$$

13. torsion

$$\tau = \frac{\vec{v}\times\vec{a}\cdot\vec{j}}{|\vec{v}\times\vec{a}|^2} = \frac{(4t^2,-4t,2)\cdot(0,0,4)}{[2(1+2t^2)]^2} = \frac{8}{4(1+2t^2)^2} = \frac{2}{(1+2t^2)^2}$$

14. tangential acceleration (use 2 methods)

$$a_T = \vec{a}\cdot\hat{T} = (0,2,4t)\cdot\frac{1}{1+2t^2}(1,2t,2t^2) = \frac{1}{1+2t^2}(4t+8t^3) = \frac{4t}{1+2t^2}(1+2t^2) = 4t$$

15. $a_T = \frac{d}{ds}\frac{ds}{dt} = \frac{d}{ds}(1+2t^2) = 4t$

16. normal acceleration (use 2 methods)

$$a_N = \vec{a}\cdot\hat{N} = (0,2,4t)\cdot\frac{1}{1+2t^2}(-2t,1-2t^2,2t) = \frac{1}{1+2t^2}(2(1-2t^2)+4t2t) = \frac{1}{1+2t^2}(2-4t^2+8t^2)$$

$$= \frac{1}{1+2t^2}(2+4t^2) = 2$$

17. $a_N = \kappa|\vec{v}|^2 = \frac{2}{(1+2t^2)^2}(1+2t^2)^2 = 2$