Name________________________ ID______________

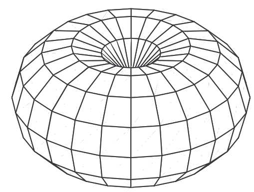
1-2	/10
3	/10
4	/10
Total	/30

MATH 251 Quiz 6 Fall 2005

Sections 503 Solutions P. Yasskin

Multiple Choice: (5 points each)

1. (5 points) Which of the following integrals will give the volume of the donut given in spherical coordinates by $\rho = \sin\varphi$.



a. $\int_0^{2\pi}\int_0^{\pi}\int_0^{\sin\varphi} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$ Correct Choice

b. $\int_0^{\pi}\int_0^{2\pi}\int_0^{\sin\varphi} \rho^2 \cos\varphi \, d\rho \, d\varphi \, d\theta$

c. $\int_0^{\pi}\int_0^{2\pi}\int_0^{1} \sin\varphi \, d\rho \, d\varphi \, d\theta$

d. $\int_0^{2\pi}\int_0^{\pi}\int_0^{1} \sin\varphi \, \rho^2 \cos\varphi \, d\rho \, d\varphi \, d\theta$

e. $\int_0^{\pi}\int_0^{2\pi}\int_0^{\sin\varphi} 1 \, d\rho \, d\varphi \, d\theta$

$$V = \iiint 1 \, dV = \int_0^{2\pi}\int_0^{\pi}\int_0^{\sin\varphi} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

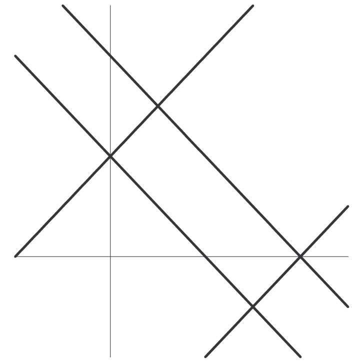
2. (5 points) A mass is pushed along the curve $\vec{r}(t) = (t^2, t^3)$ by the force $\vec{F} = (y, x)$ from the point $(0,0)$ to the point $(4,8)$. Find the work done by the force.

a. 2
b. 4
c. 8
d. 16
e. 32 Correct Choice

$$\vec{v} = \frac{d\vec{r}}{dt} = (2t, 3t^2) \qquad \vec{F}(\vec{r}(t)) = (t^3, t^2)$$

$$W = \int \vec{F} \cdot d\vec{s} = \int_0^2 \vec{F}(\vec{r}(t)) \cdot \vec{v} \, dt = \int_0^2 (2t^4 + 3t^4) \, dt = \int_0^2 5t^4 \, dt = t^5 \Big|_0^2 = 32$$

3. (10 points) Compute $\iint (x+y)\,dx\,dy$ over the region bounded by the lines
$y = x-2, \quad y = x+1, \quad y = 1-x,$ and $y = 2-x.$



Use curvilinear coordinates.
Half credit for rectangular coordinates.

$u = y - x$
$v = y + x$

Boundaries: $u = -2 \quad u = 1$
$v = 1 \quad v = 2$

Solve for x and y: $u + v = 2y \qquad y = \frac{u+v}{2} \qquad x = \frac{v-u}{2}$

$v - u = 2x \qquad x = \frac{v-u}{2} \qquad y = \frac{u+v}{2}$

Jacobian: $$J = \left| \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix} \right| = \left| \begin{vmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix} \right| = \left| -\frac{1}{4} - \frac{1}{4} \right| = \left| -\frac{1}{2} \right| = \frac{1}{2}$$

Integrand: $x + y = \frac{v-u}{2} + \frac{u+v}{2} = v$

$$\iint (x+y)\,dx\,dy = \int_1^2 \int_{-2}^1 v\,\frac{1}{2}\,du\,dv = \int_1^2 v\,\frac{1}{2}u\Big|_{u=-2}^{1} dv = \int_1^2 v\,\frac{3}{2}\,dv = \frac{3v^2}{4}\Big|_{v=1}^{2} = \frac{9}{4}$$

4. (10 points) The temperature in a box is $T = 100xyz\,°\mathrm{C}$. A wire temperature probe has the shape of the curve $\vec{r}(t) = \left(t, t^2, \frac{2}{3}t^3\right)$ for $0 \le t \le 1$. Find the average temperature along the probe given by $T_{\text{ave}} = \frac{\int T\,ds}{\int ds}$. HINT: Factor inside the square root.

$\vec{v} = (1, 2t, 2t^2) \qquad |\vec{v}| = \sqrt{1 + 4t^2 + 4t^4} = \sqrt{(1+2t^2)^2} = 1 + 2t^2$

$$\int ds = \int_0^1 |\vec{v}|\,dt = \int_0^1 (1+2t^2)\,dt = \left[t + \frac{2t^3}{3} \right]_0^1 = \frac{5}{3}$$

$$T = 100xyz = 100 \cdot t \cdot t^2 \cdot \frac{2}{3}t^3 = \frac{200}{3}t^6$$

$$\int T\,ds = \int_0^1 \frac{200}{3}t^6(1+2t^2)\,dt = \frac{200}{3}\int_0^1 (t^6 + 2t^8)\,dt = \frac{200}{3}\left[\frac{t^7}{7} + \frac{2t^9}{9} \right]_0^1 = \frac{200}{3}\left(\frac{1}{7} + \frac{2}{9} \right) = \frac{4600}{189}$$

$$T_{\text{ave}} = \frac{\int T\,ds}{\int ds} = \frac{4600}{189} \cdot \frac{3}{5} = \frac{4600}{315} \approx 014.6°\mathrm{C}$$