1-9	/45
10	/20
11	/15
12	/10
13	/15
Total	/105

Name______________________________ ID______________________

MATH 251 Exam 1 Spring 2006

Sections 506 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. A fly travels along the path $\vec{r}(t) = (3\sin t, -4\sin t, 5\cos t)$.
Find the arc length traveled by the fly between $(3,-4,0)$ and $(0,0,-5)$.

a. $\dfrac{\pi}{2}$

b. π

c. $\dfrac{3\pi}{2}$

d. 2π

e. $\dfrac{5\pi}{2}$

2. Find the unit binormal $\hat{B}$ to the curve $\vec{r}(t) = (3\sin t, -4\sin t, 5\cos t)$.

a. $\left(\frac{3}{5}, \frac{-4}{5}, 0\right)$

b. $\left(\frac{3}{5}, \frac{4}{5}, 0\right)$

c. $\left(\frac{4}{5}, \frac{-3}{5}, 0\right)$

d. $\left(\frac{4}{5}, \frac{3}{5}, 0\right)$

e. $\left(\frac{-3}{5}, \frac{4}{5}, 0\right)$

3. Find the vector projection of the vector $\vec{u} = \langle 1,-2,2\rangle$ onto the vector $\vec{v} = \langle -1,1,1\rangle$.

a. $\left\langle \frac{-1}{3}, \frac{1}{3}, \frac{1}{3} \right\rangle$

b. $\left\langle \frac{1}{3}, \frac{-1}{3}, \frac{-1}{3} \right\rangle$

c. $\left\langle \frac{1}{9}, \frac{-1}{9}, \frac{-1}{9} \right\rangle$

d. $\left\langle \frac{-1}{9}, \frac{2}{9}, \frac{-2}{9} \right\rangle$

e. $\left\langle \frac{1}{9}, \frac{-2}{9}, \frac{2}{9} \right\rangle$

4. Find the plane parallel to $4x - 3y + 2z = 5$ which passes through the point $P = (1,2,3)$.
What is the z-intercept of this plane?

a. 2

b. 4

c. 6

d. 8

e. 10

5. The plot at the right is the contour plot of which function?

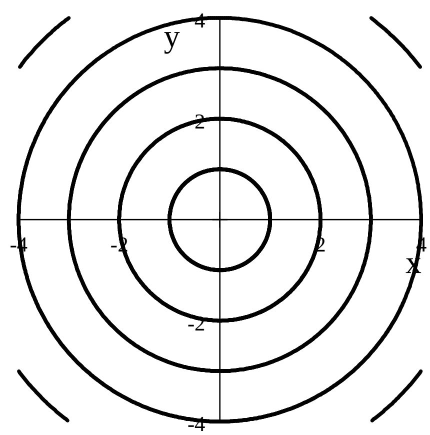
a. The hyperbolic paraboloid $z = x^2 - y^2$

b. The hyperbolic paraboloid $z = xy$

c. The hyperboloid $z = \sqrt{1 + x^2 + y^2}$

d. The cone $z = \sqrt{(x-1)^2 + y^2}$

e. The elliptic paraboloid $z = x^2 + (y-1)^2$



6. If $r = \sqrt{x^2 + y^2}$, find $\dfrac{\partial^2 r}{\partial x \partial y}(3,4)$.

a. $\dfrac{12}{5}$

b. $\dfrac{12}{25}$

c. $\dfrac{12}{125}$

d. $\dfrac{-12}{125}$

e. $\dfrac{-12}{5}$

7. The graph of a function $z = f(x,y)$ passes through the point $(2,4,8)$ and has $f_x(2,4) = -3$ and $f_y(2,4) = 5$. Use the linear approximation to approximate $f(2.2,3.9)$

a. -1.1

b. 1.1

c. 6.9

d. 7.9

e. 9.1

8. Duke Skywater is travelling through the galaxy. At the present time he is at the point with galactic coordinates $P = (40,25,53)$ (in lightyears), and his velocity is $\vec{v} = (.1,-.2,.3)$ (in lightyears /year). He measures the polaron density to be $U = 4300$ polarons/cm^3 and its gradient to be $\vec{\nabla}U = (3,2,1)$ polarons/cm^3/lightyear. Find the rate $\dfrac{dU}{dt}$ at which Duke sees the polaron density changing (in polarons/cm^3/year).

a. 0.2

b. 1.0

c. 223

d. 223.2

e. 224

9. The density of carbon monoxide in a room is given by $\delta = z\ln(9 + x^2 - y^2)$. If you start at the point $(3,4,2)$, in what direction should you move to ***decrease*** the carbon monoxide density as fast as possible?

a. $(-6,-8,-\ln 2)$

b. $(-6,8,-\ln 2)$

c. $(6,-8,-\ln 2)$

d. $(-6,8,\ln 2)$

e. $(6,-8,\ln 2)$

Work Out: (Points indicated. Part credit possible. Show all work.)

10. Find the equation of the plane tangent to each of the following surfaces:

a. $\left(10\text{ points}\right)$ $z = f(x,y) = xe^{xy}$ at $(x,y) = (2,0)$

b. $\left(10\text{ points}\right)$ $xe^{yz} + ye^{xz} = 5$ at $(x,y,z) = (2,3,0)$

11. (15 points) The radius of a cylinder is currently $r = 5$ cm and its height is $h = 10$ cm. Its radius is increasing at $\frac{dr}{dt} = 0.3 \frac{\text{cm}}{\text{sec}}$ and its height is decreasing at $\frac{dh}{dt} = -0.2 \frac{\text{cm}}{\text{sec}}$. Is its volume increasing or decreasing and at what rate?

12. (10 points) If $z = x^2 y^3$ where $x = x(u, v)$ and $y = y(u, v)$ satisfy

$$x(3,4) = 2 \qquad \frac{\partial x}{\partial u}(3,4) = 5 \qquad \frac{\partial x}{\partial v}(3,4) = 6$$

$$y(3,4) = 1 \qquad \frac{\partial y}{\partial u}(3,4) = 7 \qquad \frac{\partial y}{\partial v}(3,4) = 8$$

Find $\left.\frac{\partial z}{\partial v}\right|_{(u,v)=(3,4)}$.

13. $(15$ points$)$ Find the point on the ellipsoid $\frac{x^2}{16}+\frac{y^2}{4}+z^2=3$ at which the function $f=-x-2y+4z$ is a minimum.

You may use any method but Lagrange multipliers is easier than eliminating a variable.