1-9	/45
10	/15
11	/10
12	/20
13	/15
Total	/105

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MATH 251 Exam 2 Spring 2006

Sections 506 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Compute $\iint xy^2\,dx\,dy$ over the region bounded by $x = y^2$ and $x = 4$.

a. $\frac{256}{21}$

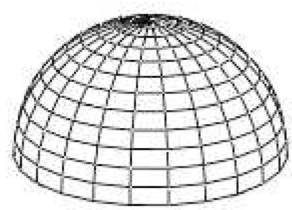
b. $\frac{512}{21}$

c. $\frac{160}{21}$

d. $\frac{272}{15}$

e. $\frac{544}{15}$

2. If $\vec{F} = (xz^2, yz^2, z^3)$, compute $\iiint \vec{\nabla} \cdot F\,dV$ over the solid hemisphere $0 \le z \le \sqrt{4 - x^2 - y^2}$.



a. $\frac{64\pi}{3}$

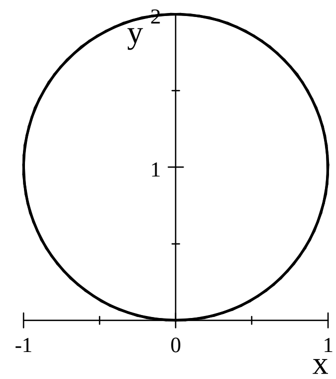
b. $\frac{64\pi}{15}$

c. $\frac{32\pi}{15}$

d. $\frac{320\pi}{3}$

e. $\frac{640\pi}{3}$

3. Compute $\iint \frac{1}{y}\, dA$ over the region inside the circle given in polar coordinates by $r = 2\sin\theta$.



a. $\frac{\pi}{4}$
b. $\frac{\pi}{2}$
c. π
d. 2π
e. undefined

4. Find the mass of the solid inside the cylinder $x^2 + y^2 = 4$ between $z = 0$ and $z = 3 - \sqrt{x^2 + y^2}$ if the density is $\delta = \sqrt{x^2 + y^2}$.

a. $\frac{\pi}{4}$
b. $\frac{\pi}{2}$
c. π
d. 2π
e. 8π

5. Find the z-component of the center of mass of the solid inside the cylinder $x^2 + y^2 = 4$ between $z = 0$ and $z = 3 - \sqrt{x^2 + y^2}$ if the density is $\delta = \sqrt{x^2 + y^2}$.

a. $\frac{32\pi}{5}$
b. $\frac{16\pi}{5}$
c. $\frac{4}{5}$
d. $\frac{5}{32\pi}$
e. $\frac{5}{4}$

6. Compute $\int_0^2 \int_y^2 e^{x^2}\, dx\, dy$

a. $\frac{1}{2}(1 - e^{-2})$
b. $\frac{1}{2}e^{-2}$
c. $\frac{1}{2}(e^2 - 1)$
d. $\frac{1}{2}e^4$
e. $\frac{1}{2}(e^4 - 1)$

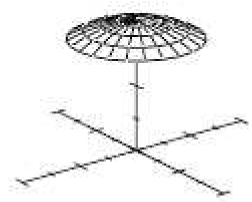
7. Compute $\oint \vec{F} \cdot d\vec{s}$ for the vector field $\vec{F} = (-y, x)$ along the curve $\vec{r}(t) = (\sin t \cos t, \sin^2 t)$ for $0 \le t \le \pi$. HINT: Factor $\vec{F} \cdot \vec{v}$.

a. $\frac{\pi}{4}$
b. $\frac{\pi}{2}$
c. π
d. 2π
e. 4π

8. If $f = \sin(xyz)$ compute $\vec{\nabla} \cdot \vec{\nabla} f$.

a. 0
b. $\sin(xyz)(-y^2z^2, x^2z^2, -x^2y^2)$
c. $\sin(xyz)(-y^2z^2, -x^2z^2, -x^2y^2)$
d. $-(y^2z^2 + x^2z^2 + x^2y^2)\sin(xyz)$
e. $-(y^2z^2 - x^2z^2 + x^2y^2)\sin(xyz)$

9. Find the area of the polar ice cap given in spherical coordinates by $0 \le \varphi \le \frac{\pi}{6}$ given that the radius of the earth is 4000 miles. HINT: Parametrize the piece of the sphere and find the normal vector.



a. $4000^2\pi\left(2 - \sqrt{3}\right)$

b. $4000^2\left(1 - \frac{\sqrt{3}}{2}\right)$

c. $4000^2\left(2 - \sqrt{3}\right)$

d. $4000^2\pi\left(1 - \frac{\sqrt{3}}{2}\right)$

e. $4000^2\pi$

Work Out: (Part credit possible. Show all work.)

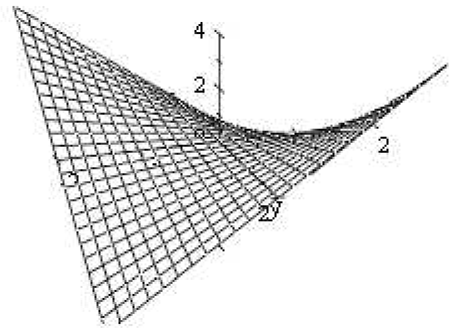
10. $\left(15 \text{ points}\right)$ The average of a function f on a curve C is $f_{\text{ave}} = \frac{1}{L}\int_C f\,ds$ where L is the length of the curve. Find the average temperature T_{ave} on the curve $\vec{r}(t) = \left(t, t^2, \frac{2}{3}t^3\right)$ between $(0,0,0)$ and $\left(1,1,\frac{2}{3}\right)$ if the temperature is $T = xy^2 + 3yz$.

11. $\left(10 \text{ points}\right)$ Compute $\iiint x\,dV$ over the triangular pyramid with vertices $(0,0,0)$, $(1,0,0)$, $(0,2,0)$, and $(0,0,4)$.

12. $\left(20 \text{ points}\right)$ Consider the saddle $z = xy$

which may be parametrized by $\vec{R}(u,v) = (u,v,uv)$.

Compute $\iint \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ for $\vec{F} = (-yz, xz, 0)$ over the piece of the saddle with $0 \le x \le 2$ and $0 \le y \le 3$ with normal pointing up.

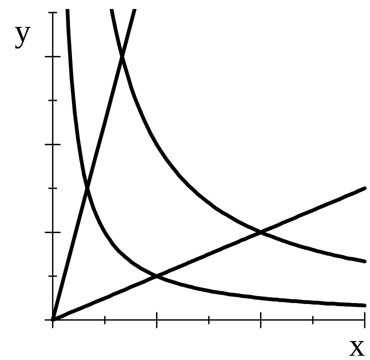


HINT: First compute each of the following:

$\vec{\nabla} \times \vec{F}$, $\vec{\nabla} \times \vec{F}\left(\vec{R}(u,v)\right)$, $\vec{e}_u$, $\vec{e}_v$, $\vec{N}$, $\vec{\nabla} \times \vec{F} \cdot \vec{N}$

13. (15 points) Compute $\iint_R \frac{y}{x}\,dx\,dy$ over the diamond shaped region R bounded by

$$y = \frac{1}{x}, \qquad y = \frac{4}{x}, \qquad y = x, \qquad y = 9x$$



FULL CREDIT for integrating in the curvilinear coordinates (u,v) where $u^2 = xy$ and $v^2 = \frac{y}{x}$.
(Solve for x and y.)

HALF CREDIT for integrating in rectangular coordinates.