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MATH 251 Quiz 1 Spring 2006

Sections 506 Solutions P. Yasskin

Multiple Choice & Work Out: (5 points each)

1. A triangle has vertices $P = (4,1,2)$, $Q = (2,1,4)$ and $R = (2,1,7)$. Find the angle at vertex Q.

a. $\frac{\pi}{4}$

b. $\frac{-\pi}{4}$

c. $\frac{\pi}{2}$

d. $\frac{-\pi}{2}$

e. $\frac{3\pi}{4}$ Correct Choice

$\overrightarrow{QP} = P - Q = (2,0,-2)$ $\overrightarrow{QR} = R - Q = (0,0,3)$ $\overrightarrow{QP} \cdot \overrightarrow{QR} = -6$

$\left|\overrightarrow{QP}\right| = \sqrt{4+4} = 2\sqrt{2}$ $\left|\overrightarrow{QR}\right| = \sqrt{9} = 3$

$$\cos\theta = \frac{\overrightarrow{QP} \cdot \overrightarrow{QR}}{\left|\overrightarrow{QP}\right|\left|\overrightarrow{QR}\right|} = \frac{-6}{2\sqrt{2}\,3} = \frac{-1}{\sqrt{2}} \quad \Rightarrow \quad \theta = 135^\circ = \frac{3\pi}{4}$$

2. A triangle has vertices $P = (4,1,2)$, $Q = (2,1,4)$ and $R = (2,1,7)$. Find the area of the triangle.

a. 3 Correct Choice

b. 6

c. $6\sqrt{3}$

d. 18

e. 36

$\overrightarrow{QP} = P - Q = (2,0,-2)$ $\overrightarrow{QR} = R - Q = (0,0,3)$

$$\overrightarrow{QP} \times \overrightarrow{QR} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 0 & -2 \\ 0 & 0 & 3 \end{vmatrix} = \hat{\imath}(0) - \hat{\jmath}(6) + \hat{k}(0) = (0,-6,0)$$

$$\text{Area} = \frac{1}{2}\left|\overrightarrow{QP} \times \overrightarrow{QR}\right| = \frac{1}{2}\sqrt{36} = 3$$

3. If $\vec{u}$ points NorthWest and $\vec{v}$ points Down (toward the center of the earth), then $\vec{u} \times \vec{v}$ points

a. Up

b. SouthEast

c. SouthWest Correct Choice

d. NorthEast

e. NorthWest

Put your fingers NorthWest with the palm facing Down, your thumb points SouthWest.

4. Find the equation of the line which is perpendicular to the plane $2x - 4y + 3z = 3$ and passes through the point $(3,2,-1)$. HINT: The normal to the plane is the tangent to the line.

a. $(x,y,z) = (3 + 2t, 2 + 4t, -1 + 3t)$

b. $(x,y,z) = (3 + 2t, 2 - 4t, -1 + 3t)$ Correct Choice

c. $(x,y,z) = (2 + 3t, 4 + 2t, 3 - t)$

d. $(x,y,z) = (2 + 3t, -4 + 2t, 3 - t)$

e. $(x,y,z) = (2 + 3t, 4 - 2t, 3 - t)$

The normal to the plane is $\vec{N} = (2,-4,3)$. So the tangent vector to the line is $\vec{v} = (2,-4,3)$.
A point on the line is $P = (3,2,-1)$. So the line is $X = P + t\vec{v} = (3 + 2t, 2 - 4t, -1 + 3t)$.

5. Find the point where the line $(x,y,z) = (1 - t, 2 + 2t, -3 + 3t)$ intersects the plane $3x - 2y + z = 4$.

Substitute the line into the plane and solve for t:

$3(1 - t) - 2(2 + 2t) + (-3 + 3t) = 4 \qquad -4 - 4t = 4 \qquad -4t = 8 \qquad t = (-2)$

Substitute back into the line:

$(x,y,z) = (1 - (-2), 2 + 2(-2), -3 + 3(-2)) = (3,-2,-9)$