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MATH 251 Quiz 2 Spring 2006

Sections 506 Solutions P. Yasskin

Total	/34

All Work Out: (2 points each, includes 9 points extra credit)

Consider the parametric curve with position vector $\vec{r} = \left(6t, 3\sqrt{2}\,t^2, 2t^3\right)$.
Compute each of the following:

1. velocity
$\vec{v} = \left(6, 6\sqrt{2}\,t.6t^2\right)$

2. acceleration
$\vec{a} = \left(0, 6\sqrt{2}, 12t\right)$

3. jerk
$\vec{j} = (0, 0, 12)$

4. length of velocity Simplify. (Note the quantity in the square root is a perfect square.)

$$|\vec{v}| = \sqrt{36 + 72t^2 + 36t^4} = 6\sqrt{(1+t^2)^2} = 6 + 6t^2$$

5. speed
$\dfrac{ds}{dt} = |\vec{v}(t)| = 6 + 6t^2$

6. arclength between the points $(0,0,0)$ and $\left(6\sqrt{2}, 6\sqrt{2}, 4\sqrt{2}\right)$

$$L = \int_{(0,0,0)}^{\left(6\sqrt{2},6\sqrt{2},4\sqrt{2}\right)} ds = \int_0^{\sqrt{2}} |\vec{v}|\,dt = \int_0^{\sqrt{2}} (6 + 6t^2)\,dt = [6t + 2t^3]_0^1 = 6\sqrt{2} + 4\sqrt{2} = 10\sqrt{2}$$

7. unit tangent vector

$$\hat{T} = \frac{\vec{v}}{|\vec{v}|} = \frac{1}{6+6t^2}\left(6, 6\sqrt{2}\,t.6t^2\right) = \frac{1}{1+t^2}\left(1, \sqrt{2}\,t.t^2\right) = \left(\frac{1}{1+t^2}, \frac{\sqrt{2}\,t}{1+t^2}, \frac{t^2}{1+t^2}\right)$$

8. cross product of velocity and acceleration

$$\vec{v}\times\vec{a} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 6 & 6\sqrt{2}\,t & 6t^2 \\ 0 & 6\sqrt{2} & 12t \end{vmatrix} = \hat{\imath}\left(72\sqrt{2}\,t^2 - 36\sqrt{2}\,t^2\right) - \hat{\jmath}(72t) + \hat{k}\left(36\sqrt{2}\right)$$
$$= \left(36\sqrt{2}\,t^2, -72t, 36\sqrt{2}\right) = 36\sqrt{2}\left(t^2, -\sqrt{2}\,t, 1\right)$$

9. length of cross product of velocity and acceleration

$$|\vec{v}\times\vec{a}| = 36\sqrt{2}\sqrt{t^4 + 2t^2 + 1} = 36\sqrt{2}\,(1+t^2)$$

10. unit binormal

$$\hat{B} = \frac{\vec{v}\times\vec{a}}{|\vec{v}\times\vec{a}|} = \frac{36\sqrt{2}\left(t^2,-\sqrt{2}\,t,1\right)}{36\sqrt{2}\,(1+t^2)} = \frac{1}{1+t^2}\left(t^2,-\sqrt{2}\,t,1\right) = \left(\frac{t^2}{1+t^2},\frac{-\sqrt{2}\,t}{1+t^2},\frac{1}{1+t^2}\right)$$

11. unit principal normal

$$\hat{N} = \hat{B}\times\hat{T} = \frac{1}{(1+t^2)^2}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ t^2 & -\sqrt{2}\,t & 1 \\ 1 & \sqrt{2}\,t & t^2 \end{vmatrix} = \frac{1}{(1+t^2)^2}\left[\hat{\imath}\left(-\sqrt{2}\,t^3-\sqrt{2}\,t\right)-\hat{\jmath}(t^4-1)+\hat{k}\left(\sqrt{2}\,t^3+\sqrt{2}\,t\right)\right]$$

$$= \frac{1}{(1+t^2)^2}\left(-\sqrt{2}\,t^3-\sqrt{2}\,t,1-t^4,\sqrt{2}\,t^3+\sqrt{2}\,t\right)$$

Optional: $$\hat{N} = \frac{1}{(1+t^2)^2}\left(-\sqrt{2}\,t(t^2+1),(1-t^2)(1+t^2),\sqrt{2}\,t(t^2+1)\right)$$

$$= \frac{1}{1+t^2}\left(-\sqrt{2}\,t,1-t^2,\sqrt{2}\,t\right) = \left(\frac{-\sqrt{2}\,t}{1+t^2},\frac{1-t^2}{1+t^2},\frac{\sqrt{2}\,t}{1+t^2}\right)$$

12. curvature

$$\kappa = \frac{|\vec{v}\times\vec{a}|}{|\vec{v}|^3} = \frac{36\sqrt{2}\,(1+t^2)}{(6+6t^2)^3} = \frac{\sqrt{2}}{6(1+t^2)^2}$$

13. torsion

$$\tau = \frac{\vec{v}\times\vec{a}\bullet\vec{j}}{|\vec{v}\times\vec{a}|^2} = \frac{36\sqrt{2}\left(t^2,-\sqrt{2}\,t,1\right)\bullet(0,0,12)}{\left[36\sqrt{2}\,(1+t^2)\right]^2} = \frac{36\sqrt{2}\,12}{\left(36\sqrt{2}\right)^2(1+t^2)^2} = \frac{\sqrt{2}}{6(1+2t^2)^2}$$

14. tangential acceleration (use 2 methods)

$$a_T = \vec{a}\bullet\hat{T} = \left(0,6\sqrt{2},12t\right)\bullet\frac{1}{1+t^2}\left(1,\sqrt{2}\,t.t^2\right) = \frac{1}{1+t^2}(12t+12t^3) = 12t$$

15. $a_T = \dfrac{d}{ds}\dfrac{ds}{dt} = \dfrac{d}{ds}(6+6t^2) = 12t$

16. normal acceleration (use 2 methods)

$$a_N = \vec{a}\bullet\hat{N} = \left(0,6\sqrt{2},12t\right)\bullet\frac{1}{1+t^2}\left(-\sqrt{2}\,t,1-t^2,\sqrt{2}\,t\right) = \frac{1}{1+t^2}\left(6\sqrt{2}\,(1-t^2)+12\sqrt{2}\,t^2\right)$$

$$= \frac{6\sqrt{2}}{1+t^2}(1-t^2+2t^2) = 6\sqrt{2}$$

17. $a_N = \kappa|\vec{v}|^2 = \dfrac{\sqrt{2}}{6(1+t^2)^2}(6+6t^2)^2 = 6\sqrt{2}$