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1-3	/15
4	/10
Total	/25

MATH 251 Quiz 3 Spring 2006
Sections 506 Solutions P. Yasskin

Multiple Choice: (5 points each)

1. For the function $f(x,y) = x^2 e^{xy}$ which partial derivative is incorrect?

a. $\dfrac{\partial f}{\partial x} = 2xe^{xy} + x^2 y e^{xy}$

b. $\dfrac{\partial f}{\partial y} = x^3 e^{xy}$

c. $\dfrac{\partial^2 f}{\partial x^2} = 2e^{xy} + 4xye^{xy} + x^2 y^2 e^{xy}$

d. $\dfrac{\partial^2 f}{\partial x \partial y} = 3x^2 e^{xy} + x^2 y^2 e^{xy}$ Correct Choice

e. $\dfrac{\partial^2 f}{\partial y \partial x} = 3x^2 e^{xy} + x^3 y e^{xy}$

Use chain rule and product rule: $\dfrac{\partial^2 f}{\partial x \partial y} = 3x^2 e^{xy} + x^3 y e^{xy}$

2. Find the equation of the plane tangent to $z = x^3 y^2$ at the point $(2,1,8)$.

a. $z = 12x + 16y + 8$

b. $z = 12x + 16y - 32$ Correct Choice

c. $z = -12x - 16y + 8$

d. $z = -12x - 16y + 48$

e. $z = -12x - 16y + 32$

$f(x,y) = x^3 y^2 \qquad f(2,1) = 8$

$\dfrac{\partial f}{\partial x} = 3x^2 y^2 \qquad \dfrac{\partial f}{\partial x}(2,1) = 12$

$\dfrac{\partial f}{\partial y} = 2x^3 y \qquad \dfrac{\partial f}{\partial y}(2,1) = 16$

$$z = f_{\tan}(x,y) = f(2,1) + \frac{\partial f}{\partial x}(2,1)(x-2) + \frac{\partial f}{\partial y}(2,1)(y-1) = 8 + 12(x-2) + 16(y-1)$$

$z = 12x + 16y - 32$

3. Consider a function $g(x,y)$. If $g(3,2) = 5$, $\frac{\partial g}{\partial x}(3,2) = 4$, and $\frac{\partial g}{\partial y}(3,2) = 2$, estimate $g(3.2, 1.9)$.

a. 4.0

b. 4.2

c. 4.4

d. 5.6 Correct Choice

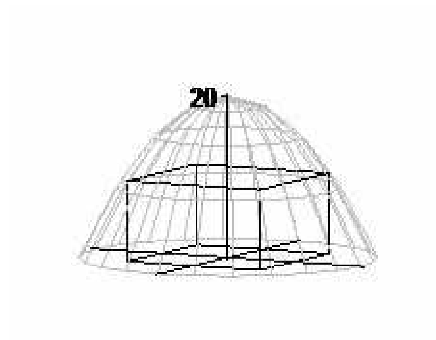
e. 6.0

$$g_{\tan}(x,y) = g(3,2) + \frac{\partial g}{\partial x}(3,2)(x-3) + \frac{\partial g}{\partial y}(3,2)(y-2) = 5 + 4(x-3) + 2(y-2)$$

$$g(3.2,1.9) \approx g_{\tan}(3.2,1.9) = 5 + 4(3.2-3) + 2(1.9-2) = 5 + 4(.2) + 2(-.1) = 5.6$$

4. $(10$ points$)$ Find the dimensions and volume of the largest rectangular box whose base is in the xy-plane, whose sides are parallel to the coordinate planes and whose top 4 vertices are on the elliptic paraboloid

$$z = 20 - x^2 - 5y^2.$$



Take x and y in the first quadrant. Then

$V = 4xyz = 4xy(20 - x^2 - 5y^2) = 80xy - 4x^3y - 20xy^3$

$V_x = 80y - 12x^2y - 20y^3 = 0$ $\quad$ $V_y = 80x - 4x^3 - 60xy^2 = 0$

$V_x = 4y(20 - 3x^2 - 5y^2) = 0$ $\quad$ $V_y = 4x(20 - x^2 - 15y^2) = 0$

If x or y is 0, then the volume is 0 and this cannot be the maximum volume.

So we solve $20 - 3x^2 - 5y^2 = 0$ and $20 - x^2 - 15y^2 = 0$.

Multiply the second equation by 3 and subtract the first equation:

$60 - 3x^2 - 45y^2 = 0$ minus $20 - 3x^2 - 5y^2 = 0$ equals $40 - 40y^2 = 0$

So: $y = 1$ $\quad$ $x^2 = 20 - 15y^2 = 5$ $\quad$ $x = \sqrt{5}$

$z = 20 - x^2 - 5y^2 = 20 - 5 - 5 = 10$

$V = 4xyz = 4 \cdot \sqrt{5} \cdot 1 \cdot 10 = 40\sqrt{5}$