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1-2	/20
3	/10
Total	/30

MATH 251 Quiz 4 Spring 2006

Sections 506 Solutions P. Yasskin

10 points each

1. Compute $\int_1^2 \int_1^x 3y\; dy\; dx$.

a. -1
b. 1
c. 2 Correct Choice
d. $\frac{7}{2}$
e. 4

$$\int_1^2 \int_1^x 3y\; dy\; dx = \int_1^2 \left[\frac{3y^2}{2} \right]_{y=1}^{x} dx = \int_1^2 \frac{3x^2}{2} - \frac{3}{2}\; dx = \left[\frac{x^3}{2} - \frac{3x}{2} \right]_1^2 = [4-3] - \left[\frac{1}{2} - \frac{3}{2} \right] = 2$$

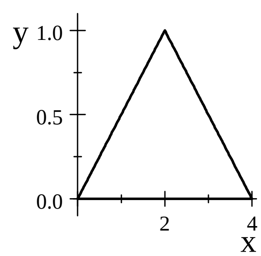
2. Find the volume under the surface $z = 2xy^2$ above the triangle with vertices $(0,0)$, $(4,0)$ and $(2,1)$.

a. $-\frac{1}{3}$
b. $\frac{1}{3}$
c. $\frac{2}{3}$
d. $\frac{7}{6}$
e. $\frac{4}{3}$ Correct Choice

$$\int_0^1 \int_{2y}^{4-2y} 2xy^2\, dx\, dy = \int_0^1 \left[x^2y^2 \right]_{x=2y}^{4-2y} dy = \int_0^1 \left[(4-2y)^2 - (2y)^2 \right] y^2\, dy$$

$$= \int_0^1 (16-16y)y^2\, dy = \int_0^1 16y^2 - 16y^3\, dy = \left[\frac{16}{3} y^3 - 4y^4 \right]_0^1$$

$$= \left[\frac{16}{3} - 4 \right] = \frac{4}{3}$$



3. Compute $\int_0^1 \int_{y^2}^1 y^3 \sin(x^3)\, dx\, dy$.   HINT: Reverse the order of integration.

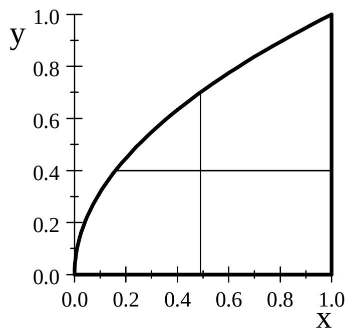
Plot the region:

Originally:

$0 \le y \le 1 \qquad y^2 \le x \le 1$

Finally:

$0 \le x \le 1 \qquad 0 \le y \le \sqrt{x}$



$$\int_0^1 \int_{y^2}^1 y^3 \sin(x^3)\, dx\, dy = \int_0^1 \int_0^{\sqrt{x}} y^3 \sin(x^3)\, dy\, dx = \int_0^1 \left[\frac{y^4}{4} \right]_{y=0}^{\sqrt{x}} \sin(x^3)\, dx = \int_0^1 \frac{x^2}{4} \sin(x^3)\, dx$$

Now let $u = x^3$. Then $du = 3x^2\, dx$. Or $\frac{1}{3}\, du = x^2\, dx$.

Also if $x = 0$ then $u = 0$ and if $x = 1$ then $u = 1$. So

$$\int_0^1 \int_{y^2}^1 y^3 \sin(x^3)\, dx\, dy = \int_0^1 \frac{1}{12} \sin(u)\, du = \left[\frac{-1}{12} \cos u \right]_0^1 = \frac{-1}{12} \cos 1 + \frac{1}{12}$$