Name________________________ ID__________________

1-9	/45	12	/12
10	/12	13	/12
11	/12	14	/12
Total	/105		

MATH 251 Exam 2 Fall 2006

Sections 507 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Compute $\int_0^2 \int_0^z \int_0^{xz} 30x\,dy\,dx\,dz$.

a. 4

b. 8

c. 16

d. 32

e. 64

2. Compute $\int_0^2 \int_{y^2}^4 y\sin(x^2)\,dx\,dy$ by interchanging the order of integration.

a. $\frac{-\cos 16}{2}$

b. $\frac{1-\cos 16}{4}$

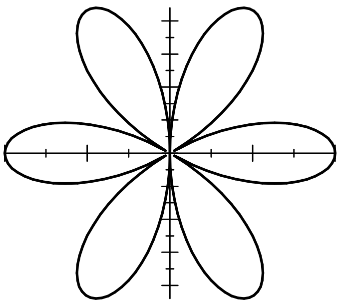
c. $\frac{\cos 16 - 1}{2}$

d. $\frac{\cos 16}{8}$

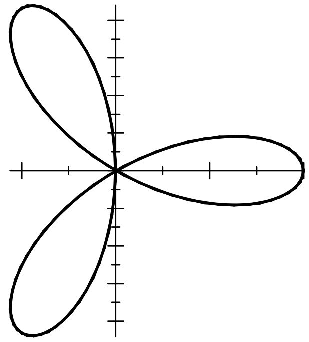
e. $\frac{\cos 16 - 1}{4}$

3. Which of the following is the polar plot of $r = \sin(3\theta)$?

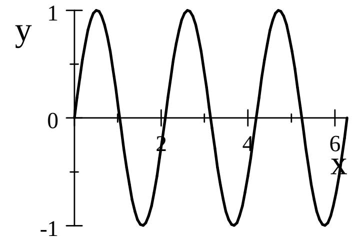
a.



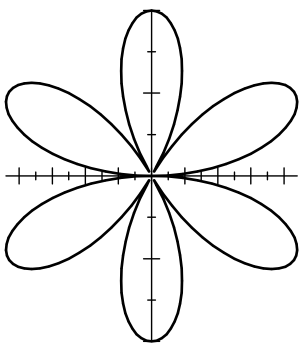
b.



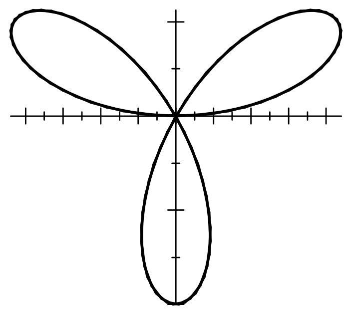
c.



d.

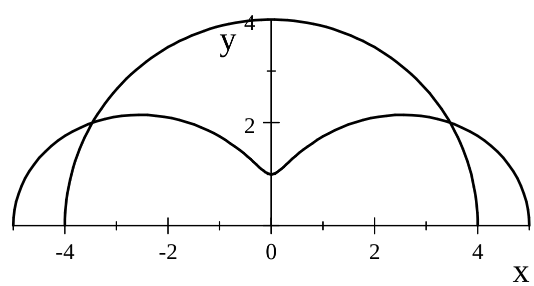


e.



4. Find the area of the region inside the circle $r = 4$ outside the polar curve $r = 3 + 2\cos(2\theta)$ with $y \geq 0$.

The area is given by the integral:



a. $A = \int_{\pi/3}^{5\pi/3} \int_{3+2\cos(2\theta)}^{4} dr\, d\theta$

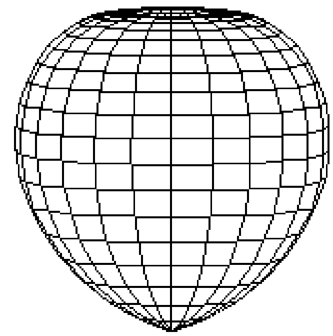
b. $A = \int_{\pi/3}^{5\pi/3} \int_{3+2\cos(2\theta)}^{4} r\, dr\, d\theta$

c. $A = \int_{\pi/6}^{5\pi/6} \int_{3+2\cos(2\theta)}^{4} dr\, d\theta$

d. $A = \int_{\pi/6}^{5\pi/6} \int_{3+2\cos(2\theta)}^{4} r\, dr\, d\theta$

e. $A = \int_{\pi/3}^{5\pi/3} \int_{4}^{3+2\cos(2\theta)} dr\, d\theta$

5. Find the volume of the apple given in spherical coordinates by $\rho = \varphi$.



The volume is given by the integral:

a. $\frac{2\pi}{3}\int_0^{\pi} \varphi^3 \sin\varphi \, d\varphi$

b. $2\pi\int_0^{2\pi} \varphi^2 \sin\varphi \, d\varphi$

c. $2\pi\int_0^{\pi} \varphi^2 \sin\varphi \, d\varphi$

d. $\pi\int_0^{2\pi} \varphi^2 \sin\varphi \, d\varphi$

e. $\pi\int_0^{\pi} \varphi^2 \sin\varphi \, d\varphi$

6. Find a scalar potential f for the vector field $\vec{F} = (y+z, x+z, x+y+2z)$.
Then evaluate $f(1,1,1) - f(0,0,0)$:

a. 1

b. 2

c. 4

d. 5

e. 7

7. Which vector field cannot be written as $\vec{\nabla} \times \vec{F}$ for any vector field $\vec{F}$.

a. $\vec{A} = (x, y, -2z)$

b. $\vec{B} = (xz, yz, z^2)$

c. $\vec{C} = (xz, yz, -z^2)$

d. $\vec{D} = (z\sin x, -yz\cos x, y\sin x)$

e. $\vec{E} = (x\sin y, \cos y, x\cos y)$

8. Find the total mass of a plate occupying the region between $y = x^2$ and $y = 4$ if the mass density is $\rho = y$.

a. $\frac{32}{3}$

b. $\frac{32}{9}$

c. $\frac{64}{5}$

d. $\frac{128}{5}$

e. $\frac{256}{5}$

9. Find the center of mass of a plate occupying the region between $y = x^2$ and $y = 4$ if the mass density is $\rho = y$.

a. $\left(0, \frac{20}{7}\right)$

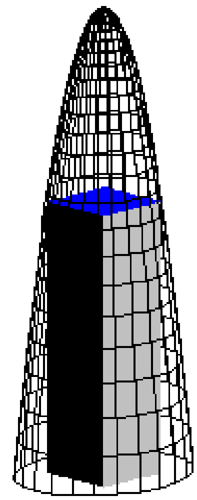
b. $\left(0, \frac{12}{5}\right)$

c. $\left(0, \frac{512}{7}\right)$

d. $\left(0, \frac{128}{5}\right)$

e. $\left(0, \frac{14}{5}\right)$

Work Out: (12 points each. Part credit possible. Show all work.)



10. Find the dimensions and volume of the largest box which sits on the xy-plane and whose upper vertices are on the elliptic paraboloid $z = 12 - 2x^2 - 3y^2$.

You do not need to show it is a maximum.
You MUST eliminate the constraint.
Do not use Lagrange multipliers.

11. A pot of water is sitting on a stove. The pot is a cylinder of radius 3 inches and height 4 inches. If the origin is located at the center of the bottom, then the temperature of the water is $T = 102 + x^2 + y^2 - z$. Find the average temperature of the water: $T_{\text{ave}} = \dfrac{\iiint T\,dV}{\iiint dV}$.

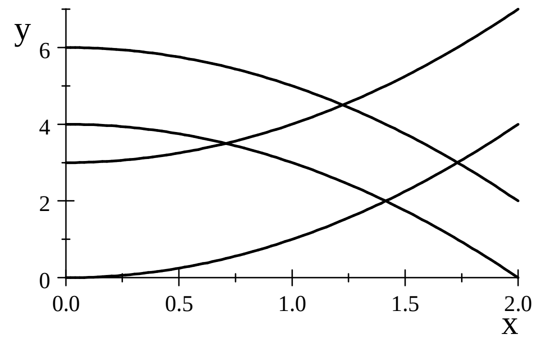
12. Compute $\iint_D x\,dx\,dy$ over the "diamond" shaped region bounded by the curves

$y = x^2 \qquad y = 3 + x^2 \qquad y = 4 - x^2 \qquad y = 6 - x^2$

Use the curvilinear coordinates

$u = y + x^2$ and $v = y - x^2$.

(Half credit for using rectangular coordinates.)



13. The sides of a cylinder C of radius 3 and height 4 may be parametrized by

$R(h,\theta) = (3\cos\theta, 3\sin\theta, h)$ for $0 \le \theta \le 2\pi$ and $0 \le h \le 4$.

Compute $\iint_C \vec{\nabla} \times \vec{F} \cdot d\vec{S}$ for $\vec{F} = (-yz^2, xz^2, z^3)$ and outward normal.

HINT: Find $\vec{e}_h$, $\vec{e}_\theta$, $\vec{N} = \vec{e}_h \times \vec{e}_\theta$, $\vec{\nabla} \times \vec{F}$ and $\left(\vec{\nabla} \times \vec{F}\right)\left(\vec{R}(h,\theta)\right)$.

14. The hemispherical surface $x^2 + y^2 + z^2 = 9$ has surface density $\rho = x^2 + y^2$.

The surface may be parametrized by $\vec{R}(\varphi, \theta) = (3\sin\varphi\cos\theta, 3\sin\varphi\sin\theta, 3\cos\varphi)$.

Find the mass and center of mass of the surface.

HINT: Find $\vec{e}_\varphi$, $\vec{e}_\theta$, $\vec{N} = \vec{e}_\varphi \times \vec{e}_\theta$, $\left|\vec{N}\right|$ and $\rho\left(\vec{R}(\varphi, \theta)\right)$.