1-10	/50
11	/15
12	/15
13	/15
14	/15
Total	/110

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MATH 251 Final Exam Fall 2006

Sections 507 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. For the curve $\vec{r}(t) = (t\cos t, t\sin t)$, which of the following is false?

a. The velocity is $\vec{v} = (\cos t - t\sin t, \sin t + t\cos t)$

b. The speed is $|\vec{v}| = \sqrt{1+t^2}$

c. The acceleration is $\vec{a} = (-2\sin t - t\cos t, 2\cos t - t\sin t)$

d. The arclength between $t = 0$ and $t = 1$ is $L = \int_0^1 t\sqrt{1+t^2}\,dt$

e. The tangential acceleration is $a_T = \dfrac{t}{\sqrt{1+t^2}}$

2. Find the plane tangent to the surface $x^2z^2 + y^4 = 5$ at the point $(2,1,1)$.

a. $2x + y + z = 6$

b. $2x + y + z = 5$

c. $x + y + 2z = 5$

d. $x - y + 2z = 3$

e. $x - y + 2z = 6$

3. Let $L = \lim_{(x,y)\to(0,0)} \dfrac{x^2 + xy^2}{x^2 + y^4}$

a. L exists and $L = 1$ by looking at the paths $y = mx$.

b. L does not exist by looking at the paths $y = x$ and $y = \sqrt{x}$.

c. L does not exist by looking at the paths $y = \sqrt{x}$ and $y = -\sqrt{x}$.

d. L does not exist by looking at the paths $x = my^2$.

e. L does not exist by looking at the paths $x = y^3$ and $x = -y^3$.

4. The point $(1,-3)$ is a critical point of the function $f = xy^2 - 3x^3 + 6y$. It is a

a. local minimum.

b. local maximum.

c. saddle point.

d. inflection point.

e. The Second Derivative Test fails.

5. The dimensions of a closed rectangular box are measured as 70 cm, 50 cm and 40 cm with a possible error of 0.2 cm in each dimension. Use differentials to estimate the maximum error in the calculated surface area of the box.

a. 8

b. 16

c. 32

d. 64

e. 128

6. Consider the quarter of the cylinder $x^2 + y^2 \le 4$ with $x \ge 0,\ \ y \ge 0$ and $0 \le z \le 8$.
Find the total mass of the quarter cylinder if the density is $\rho = e^{x^2+y^2}$.

a. $2\pi(e^4 - 1)$

b. $8\pi(e^4 - 1)$

c. $2\pi e^4$

d. $8\pi e^4$

e. 4

7. Consider the quarter of the cylinder $x^2 + y^2 \le 4$ with $x \ge 0,\ \ y \ge 0$ and $0 \le z \le 8$.
Find the z- component of the center of mass of the quarter cylinder if the density is $\rho = e^{x^2+y^2}$.

a. $2\pi(e^4 - 1)$

b. $8\pi(e^4 - 1)$

c. $2\pi e^4$

d. $8\pi e^4$

e. 4

8. Compute the line integral $\int y\,dx - x\,dy$ counterclockwise around the semicircle $x^2 + y^2 = 9$ from $(3,0)$ to $(-3,0)$. (HINT: Parametrize the curve.)

a. -9π

b. -3π

c. π

d. 3π

e. 9π

9. Compute the line integral $\int \vec{F} \cdot d\vec{s}$ for the vector field $\vec{F} = (y, x)$ along the curve $\vec{r}(t) = \left(e^{\cos(t^2)}, e^{\sin(t^2)}\right)$ for $0 \le t \le \sqrt{\pi}$. (HINT: Find a scalar potential.)

a. $e - \frac{1}{e}$

b. $\frac{1}{e} - e$

c. $\frac{2}{e}$

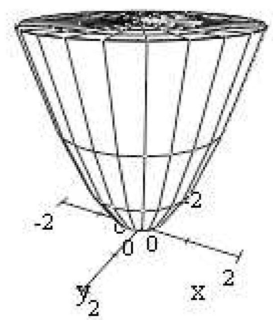
d. $2e$

e. 0

10. Consider the parabolic surface P given by $z = x^2 + y^2$ for $z \le 4$ with normal pointing up and in, the disk D given by $x^2 + y^2 \le 4$ and $z = 4$ with normal pointing up, and the volume V between them. Given that for a certain vector field $\vec{F}$ we have

$$\iiint_V \vec{\nabla} \cdot \vec{F}\, dV = 14 \quad \text{and} \quad \iint_D \vec{F} \cdot d\vec{S} = 3$$

compute $\iint_P \vec{F} \cdot d\vec{S}$.



a. 17

b. 11

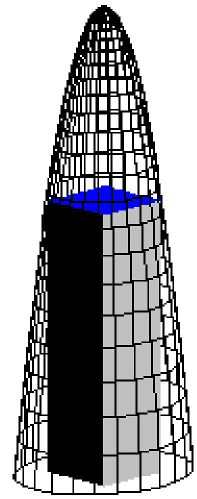
c. 8

d. -11

e. -17

Work Out: (15 points each. Part credit possible.)

11. Find the dimensions and volume of the largest box which sits on the xy-plane and whose upper vertices are on the elliptic paraboloid $z + 2x^2 + 3y^2 = 12$.



You do not need to show it is a maximum.
You MUST use the Method of Lagrange multipliers.

Half credit for the Method of Elminating the Constraint.

12. The hemisphere H given by

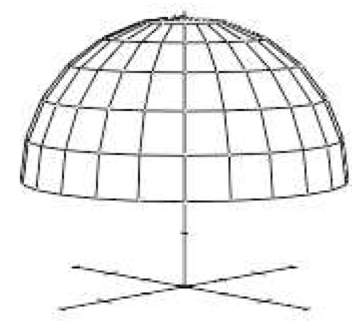
$$x^2 + y^2 + (z-2)^2 = 9 \quad \text{for} \quad z \ge 2$$

has center $(0,0,2)$ and radius 3. Verify Stokes' Theorem

$$\iint_H \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \oint_{\partial H} \vec{F} \cdot d\vec{s}$$

for this hemisphere H with normal pointing up and out

and the vector field $\vec{F} = (yz, -xz, z)$.



Be sure to check and explain the orientations. Use the following steps:

a. The hemisphere may be parametrized by

$\vec{R}(\theta,\varphi) = (3\sin\varphi\cos\theta, 3\sin\varphi\sin\theta, 2+3\cos\varphi)$

Compute the surface integral by successively finding:

$\vec{e}_\theta, \quad \vec{e}_\varphi, \quad \vec{N}, \quad \vec{\nabla}\times\vec{F}, \quad \vec{\nabla}\times\vec{F}\left(\vec{R}(\theta,\varphi)\right), \quad \iint_H \vec{\nabla}\times\vec{F}\cdot d\vec{S}$

Problem Continued

b. Parametrize the boundary circle ∂H and compute the line integral by successively finding:

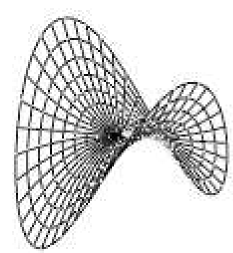
$\vec{r}(\theta)$, $\vec{v}(\theta)$, $\vec{F}(\vec{r}(\theta))$, $\oint_{\partial H} \vec{F} \cdot d\vec{s}$. Recall: $\vec{F} = (yz, -xz, z)$

13. The spider web at the right is the graph of the hyperbolic paraboloid $z = xy$.

It may be parametrized as

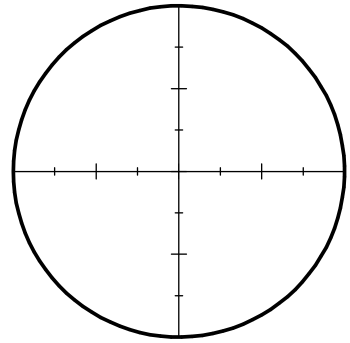
$$\vec{R}(r,\theta) = (r\cos\theta, r\sin\theta, r^2\sin\theta\cos\theta).$$

Find the area of the web for $r \le \sqrt{3}$.



14. Green's Theorem states:

$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\,dy = \oint_{\partial R} P\,dx + Q\,dy$$



Verify Green's Theorem for the functions

$P = -x^2 y$ and $Q = xy^2$

on the region inside the circle $x^2 + y^2 = 16$.

Use the following steps:

a. Compute $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$.

Then compute $\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\,dy$ by converting to polar coordinates.

b. Parametrize the boundary circle.

Compute P, Q, dx and dy on the boundary curve.

Then compute $\oint_{\partial R} P\,dx + Q\,dy$ around the boundary.