Name______________________ ID______________

1-3	/15
4	/ 5
5	/ 5
Total	/25

MATH 251 Quiz 4 Fall 2006

Sections 507 Solutions P. Yasskin

Multiple Choice & Work Out: (5 points each)

1. Compute $\vec{\nabla}\times\vec{F}$ if $\vec{F} = (x+y+z, yz+xz+xy, xyz)$.

a. 0

b. $1+x+z+xy$

c. $(1, x+z, xy)$

d. $(1, -x-z, xy)$

e. $(xz-x-y, 1-yz, y+z-1)$ Correct Choice

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ x+y+z & yz+xz+xy & xyz \end{vmatrix}$$

$$= \hat{\imath}\left[\partial_y(xyz) - \partial_z(yz+xz+xy)\right] - \hat{\jmath}\left[\partial_x(xyz) - \partial_z(x+y+z)\right] + \hat{k}\left[\partial_x(yz+xz+xy) - \partial_y(x+y+z)\right]$$

$$= \hat{\imath}(xz-x-y) - \hat{\jmath}(yz-1) + \hat{k}(y+z-1)$$

2. Compute $\vec{\nabla}\cdot\vec{F}$ if $\vec{F} = (x+y+z, yz+xz+xy, xyz)$.

a. 0

b. $1+x+z+xy$ Correct Choice

c. $(1, x+z, xy)$

d. $(1, -x-z, xy)$

e. $(xz-x-y, 1-yz, y+z-1)$

$$\vec{\nabla}\cdot\vec{F} = \partial_x(x+y+z) + \partial_y(yz+xz+xy) + \partial_z(xyz) = 1+x+z+xy$$

3. Compute $\vec{\nabla}\cdot\vec{\nabla}\times\vec{F}$ if $\vec{F} = (x+y+z, yz+xz+xy, xyz)$.

a. 0 Correct Choice

b. $1+x+z+xy$

c. $(1, x+z, xy)$

d. $(1, -x-z, xy)$

e. $(xz-x-y, 1-yz, y+z-1)$

$\vec{\nabla}\cdot\vec{\nabla}\times\vec{F} = 0$ ALWAYS

4. A rectangular box sits on the xy-plane with its upper vertices on the elliptic paraboloid $z = 16 - x^2 - 4y^2$. Find the dimensions and volume of the largest such box.

Maximize $V = (2x)(2y)z = 4xyz$ subject to $z = 16 - x^2 - 4y^2$.

Maximize $V = 4xy(16 - x^2 - 4y^2) = 64xy - 4x^3y - 16xy^3$.

$V_x = 64y - 12x^2y - 16y^3 = 4y(16 - 3x^2 - 4y^2) = 0$

$V_y = 64x - 4x^3 - 48xy^2 = 4x(16 - x^2 - 12y^2) = 0$

To have a non-zero volume, we must have $x \neq 0$ and $y \neq 0$.

So we must solve $\left\{ \begin{array}{ll} 16 - 3x^2 - 4y^2 = 0 & \text{eq 1} \\ 16 - x^2 - 12y^2 = 0 & \text{eq 2} \end{array} \right\}$

$3 \times (\text{eq 1}) - (\text{eq 2})$ is: $32 - 8x^2 = 0$ or $x^2 = 4$ or $x = 2$

Substitute into eq 1: $16 - 12 - 4y^2 = 0$ or $4y^2 = 4$ or $y = 1$

Substitute back: $z = 16 - 2^2 - 4 \cdot 1^2$ or $z = 8$

The dimensions are: $4 \times 2 \times 8$

The volume is: $V = 4xyz = 4 \cdot 2 \cdot 1 \cdot 8 = 64$

5. Find a scalar potential for $\vec{F} = (3x^2y^2 + 2xz + 1, 2x^3y - z^2, x^2 + 2z - 2yz)$ or explain why it does not exist.

Solve $\vec{\nabla} f = \vec{F}$

$\partial_x f = 3x^2y^2 + 2xz + 1 \quad \Rightarrow \quad f = x^3y^2 + x^2z + x + g(y,z)$

$\partial_y f = 2x^3y - z^2 \quad \Rightarrow \quad 2x^3y + \partial_y g = 2x^3y - z^2 \quad \Rightarrow \quad \partial_y g = -z^2 \quad \Rightarrow \quad g = -yz^2 + h(z)$

$\Rightarrow \quad f = x^3y^2 + x^2z + x - yz^2 + h(z)$

$\partial_z f = x^2 + 2z - 2yz \quad \Rightarrow \quad x^2 - 2yz + \dfrac{dh}{dz} = x^2 + 2z - 2yz \quad \Rightarrow \quad \dfrac{dh}{dz} = 2z \quad \Rightarrow \quad h = z^2$

Therefore $f = x^3y^2 + x^2z + x - yz^2 + z^2$