1-8	/40	11	/15
9	/ 5	12	/15
10	/15	13	/15
Total			/105

Name________________________ ID__________________

MATH 251 Exam 2 Spring 2007

Sections 509 P. Yasskin

Multiple Choice: (5 points each. No part credit.)

1. Find the equation of the line perpendicular to the graph of

$xyz - x^2 - y^2 - z^2 = -8$ at the point $(1,2,3)$.

Where does this line intersect the xz-plane?

a. $(9,0,11)$

b. $(9,0,-5)$

c. $(-7,0,11)$

d. $(-7,0,-5)$

e. $(5,0,-1)$

2. The point $(x,y) = (1,2)$ is a critical point of the function $f(x,y) = (x^2+y^2-4)^2 - 4x - 8y$.
Use the Second Derivative Test to classify it as a

a. local minimum

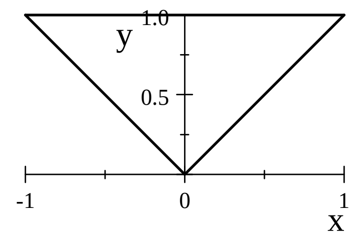
b. local maximum

c. inflection point

d. saddle point

e. Test Fails

3. Find the center of mass of the triangle with vertices $(0,0)$, $(1,1)$ and $(-1,1)$ if the mass density is $\rho = y$.



a. $\left(0, \frac{4}{5}\right)$

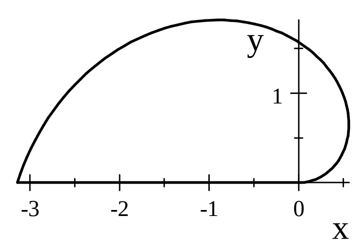
b. $\left(0, \frac{3}{4}\right)$

c. $\left(0, \frac{2}{3}\right)$

d. $\left(0, \frac{1}{2}\right)$

e. $\left(0, \frac{1}{3}\right)$

4. Compute $\iint (x^2 + y^2)\, dA$ over the region bounded by the polar curve $r = \theta$ and the x-axis.



a. $\frac{\pi^3}{6}$

b. $\frac{\pi^3}{9}$

c. $\frac{\pi^4}{12}$

d. $\frac{\pi^4}{16}$

e. $\frac{\pi^5}{20}$

5. Find the mass of the $1/8$ of the solid sphere $x^2 + y^2 + z^2 \le 16$ in the first octant if the mass density is $\delta = z$.

a. π

b. 4π

c. 8π

d. 16π

e. 64π

6. Find the volume of the solid between the cone $z = 2\sqrt{x^2 + y^2}$ and the paraboloid $z = 8 - x^2 - y^2$.
HINT: Find the radius where the cone and paraboloid intersect.

a. $\dfrac{10\pi}{3}$

b. $\dfrac{20\pi}{3}$

c. $\dfrac{40\pi}{3}$

d. $\dfrac{80\pi}{3}$

e. 30π

7. Compute $\iint_S \vec{F} \cdot d\vec{S}$ for the vector field $\vec{F} = (z, z, x + y)$ over the surface S which is parametrized by $\vec{R}(u,v) = (u + v, u - v, uv)$ for $0 \le u \le 2$ and $0 \le v \le 3$ and oriented along $\vec{N} = \vec{e}_u \times \vec{e}_v$.

a. 60

b. 12

c. 0

d. -12

e. -60

8. Compute $\iint_C \vec{F} \cdot d\vec{S}$ for the vector field $\vec{F} = (xz, yz, z^2)$ over the cylindrical surface $x^2 + y^2 = 9$ for $0 \le z \le 2$ oriented outward.

a. 36π

b. $\frac{124}{3}\pi$

c. 21π

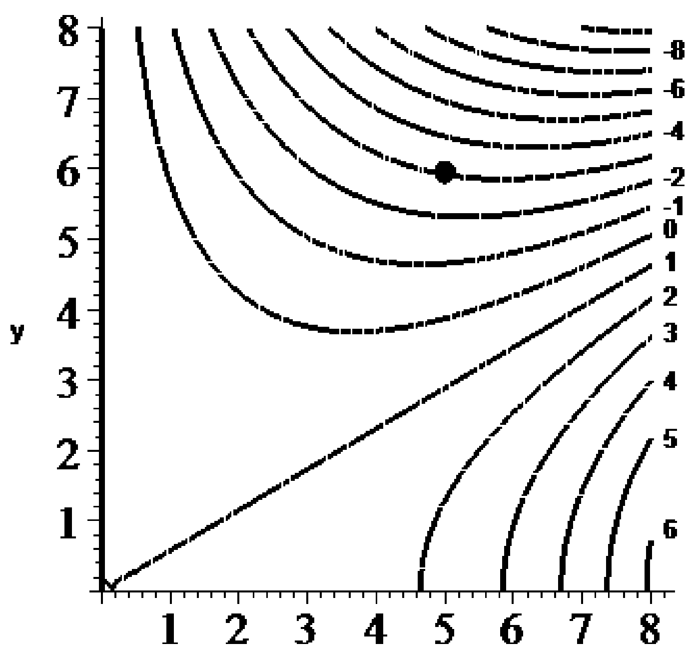
d. $\frac{62}{3}\pi$

e. 18π

Work Out: (Points indicated. Part credit possible. Show all work.)

9. (5 points) At the right is the contour plot of a function $f(x,y)$. If you **start** at the dot at $(5,6)$ and move so that your velocity is always in the direction of $\vec{\nabla} f$, the gradient of f, roughly sketch your path on the plot.

NOTE : The numbers on the right are the values of f on each level curve.

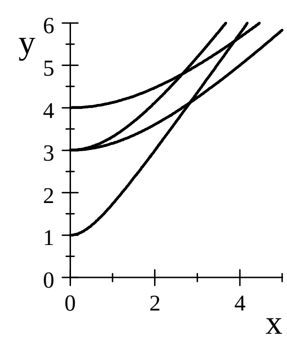


10. (15 points) An aquarium in the shape of a rectangular solid has a base made of marble which costs 6 cents per square inch, a back made of mirrored glass which costs 2 cents per square inch and a front and sides made of clear glass which costs 1 cent per square inch. There is no top. If the volume of the aquarium is 4500 cubic inches, what are the dimensions of the cheapest such aquarium?

11. (15 points) Compute $\iint xy\,dA$ over the "diamond" shaped region bounded by the curves

$$y^2 - x^2 = 9 \qquad y^2 - x^2 = 16$$
$$y^2 - 2x^2 = 1 \qquad y^2 - 2x^2 = 9$$

HINT: Let $u = y^2 - x^2$ and $v = y^2 - 2x^2$.



12. (15 points) Find the average temperature on the hemisphere surface $x^2 + y^2 + z^2 = 4$, with $z \geq 0$, if the temperature is $T = z$.

NOTE : The average of a function f is $f_{\text{ave}} = \dfrac{\iint f\, dS}{\iint dS}$. HINT: Parametrize the hemisphere.

13. (15 points) Sketch the region of integration and then compute the integral $\int_0^1 \int_{x^2}^1 x^3 \sin(y^3)\, dy\, dx$.